P. 5453. Az $r=0{,}2$ m sugarú, töltetlen fémgömb középpontjától $d=0{,}5$ m távolságra egy Q nagyságú ponttöltés helyezkedik el. Határozzuk meg (akár numerikus számítással), hogy a ponttöltéstől nézve mekkora szög alatt látszanak a fémgömb azon pontjai, ahol a felületi töltéssűrűség zérus!

(*6 pont*) Közli: *Szász Krisztián,* Budapest

Beküldési határidő: 2023. január 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS (Volume 72. No. 9. December 2022)

Problems in Mathematics

New exercises for practice – competition K (see page 542): **K. 744.** I have 1000 forints (HUF, Hungarian currency) in my pocket. If I buy two sandwiches and one soft drink then I will have as many forints remaining as the amount I should add to my 1000 in order to be able to buy three sandwiches and one soft drink. Two sandwiches and two soft drinks cost 1100 forints. What is the price of one sandwich, and what is the price of one soft drink? **K. 745.** In a game, players are collecting points. The players take turns in playing and scoring points. When a certain player is playing, he or she may get any non-negative integer of points (including 0). The points scored by a player in successive turns add up. The game terminates when the total of the points scored by the players reaches 1000 (that is, the last player may only score as many points as needed to make the total equal to 1000). The player with the largest number of points will win the game. In the case of equality, the player reaching the same score earlier will win. The player with the second largest number of points will finish in the second place, and so on. At the moment, the scores of the players are as follows: Kate has 314 points, Sam has 207 points, John has 58 points, Gillian has 31 and Joe has 0. *a*) If it is Kate's turn now, what is the minimum number of points she needs to gain in this turn in order to be certain that she will finish in the first or second place? *b*) If it is Sam's turn now, what is the minimum number of points he needs to gain in this turn in order to be certain that he will finish in the first or second place? *c*) If it is Joe's turn now, what is the minimum number of points he needs to gain in this turn in order to be certain that he will finish in the first or second place? **K. 746.** In a certain small country, citizens are charged for gas consumption as follows: in the first year of the new regulations, the first 1700 m^3 of gas costs 100 pennies/m^3, and any further consumption costs 750 pennies/m^3. From the following year onwards, the quantity sold for the reduced price is determined each year from the nationwide average consumption of the previous year. John Average lives in this country, and he used gas with these conditions for a year. Then he realized that he was paying too much, and decided to economise with the gas used in his household. He succeeded in reducing his consumption by 10% in the following year. However, since everyone was trying to save money, the mean consumption decreased by 15%. Although John Average used less gas than previously, he still had to pay more (for the whole year) than in the previous year. What may have been John's consumption during the first year? **K/C. 747.** A forty-sided polygon is divided into two polygons by one of its diagonals. The two polygons altogether have 298 fewer diagonals than the forty-gon. How many sides do these polygons have? **K/C. 748.** The integers 1 to 100 are written along the circumference of a circle. First every even number is connected to each odd number that is smaller than the even number. Then every odd number is connected to each even number that is smaller than the odd number. How many connecting lines are drawn?

New exercises for practice – competition C (see page 544): **Exercises up to grade 10: K/C. 747.** See the text at Exercises **K. K/C. 748.** See the text at Exercises **K. Exercises for everyone: C. 1743.** Seven natural numbers form an arithmetic sequence with a common difference of 30. Show that exactly one of the numbers is divisible by 7. (Proposed by *B. Bíró,* Eger) **C. 1744.** In a triangle ABC, $\angle CAB=45^\circ$ and $\angle ABC=60^\circ$. D is a point of the line segment AB. The circumscribed circle of triangle CAD passes through the orthocentre of triangle ABC. Without the use of trigonometry, find the exact value of the ratio $\frac{AD}{BD}$. (Proposed by *B. Bíró,* Eger) **C. 1745.** Solve the equation $x^2+8x-y=\frac{y-5}{y+6}$, where x, y are integers. (Proposed by *B. Bíró,* Eger) **Exercises upwards of grade 11: C. 1746.** Side AB of a square $ABCD$ is extended beyond point A by the line segment $AE=2$, and beyond point B by the line segment $BF=3$. Lines ED and FC enclose an angle of 45°. Determine the possible values of the length of the sides of the square. (Proposed by *L. Németh,* Fonyód) **C. 1747.** Let $n\geqslant 3$ be a positive integer, and let k be the sum of the digits in the number $10^n-4!$. What is the sum of the digits in the number $\frac{10^{n+1}-7}{3}$? (Proposed by *M. Szalai,* Szeged)

New exercises – competition B (see page 545): **B. 5278.** In Noname School of Nowhere, the graduating class consists of four groups, each specializing in a different subject: math, physics, chem and bio. On day, they are all sitting around a large round table in the school cafeteria: everyone has someone sitting directly opposite them, and everyone has neighbours sitting right next to them on either side. However a student is selected, it is observed that this student, the two neighbours, and the one sitting straight across the table all belong to different groups. How many students may there be in the graduating class if there are less than 20 of them? (*3 points*) (Proposed by *K. A. Kozma,* Győr) **B. 5279.** Two circles are drawn inside a right angle. One circle touches one of the arms at point A, and the other circle touches the other arm at point B. The circles also touch each other at point C. What is the measure of $\angle ACB$? (*3 points*) (Proposed by *B. Bíró,* Eger) **B. 5280.** Let $a>2$, b and c be real numbers. Consider the three statements below. (1) The equation $ax^2+bx+c=0$ has no real solution. (2) The equation $(a-1)x^2+(b-1)x+(c-1)=0$ has 1 real solution. (3) The equation $(a-2)x^2+(b-2)x+(c-2)=0$ has 2 real solutions. *a*) Given that statements (1) and (2) are true, can we conclude that statement (3) is also true? *b*) Given that statements (2) and (3) are true, can we conclude that statement (1) is also true? (*4 points*) (Proposed by *B. Hujter,* Budapest) **B. 5281.** Let $d>1$ denote a positive integer. Prove that there exists a positive integer that has the same number of factors divisible by d as factors not divisible by d. (*5 points*) (Proposed by *B. Hujter,* Budapest) **B. 5282.** In an acute-angled triangle ABC, the foot of the altitude drawn from vertex A is T. The projection of T on side AB is D, and its projection on side AC is E. Let F be the intersection of side BC and the circle ABE that is different from B. Analogously, let G be the intersection of side BC and circle ACD, different from C. Show that $TF=TG$. (*5 points*) (Proposed by *G. Kós,* Budapest) **B. 5283.** A convex quadrilateral N contains a circular disc of radius r. Show that the perimeter of N is at least $8r$. (*4 points*) (Proposed by *V. Vígh,* Sándorfalva) **B. 5284.** Let $n > 2$. Alan has selected an edge of the complete graph of $2n$ vertices. Paula wants to find out which. If she pays 1 forint (Hungarian currency, HUF) she may name any pairing of all vertices and ask whether the selected edge is contained in it. What is the minimum number of forints that Paula needs to have in her pockets in order to be certain that she can find out the selected edge by asking the appropriate questions? (*6 points*) (Proposed by *P. P. Pach,* Budapest) **B. 5285.** In an acute-angled triangle ABC, $AB = AC$. Points A', B' and C' are moving along the circumscribed circle of the triangle, so that triangle $A'B'C'$ always remain congruent to triangle ABC, and have the same orientation. Let P be the intersection of lines BB' and CC'. Show that the lines $A'P$ all pass through a certain point. (*6 points*) (Proposed by *G. Kós,* Budapest)

New problems – competition A (see page 546): **A. 839.** We are given a finite, simple, non-directed graph. Ann writes positive real numbers on each edge of the graph such that for all vertices the following is true: the sum of the numbers written on the edges incident to a given vertex is less than one. Bob wants to write non-negative real numbers on the vertices in the following way: if the number written at vertex v is v_0, and Ann's numbers on the edges incident to v are $e_1, e_2, \ldots, e_k$, and the numbers on the other endpoints of these edges are $v_1, v_2, \ldots, v_k$, then $v_0 = \sum\limits_{i=1}^{k} e_i v_i + 2022$. Prove that Bob can always number the vertices in this way regardless of the graph and the numbers chosen by Ann. (Proposed by *Boldizsár Varga,* Verőce) **A. 840.** The incircle of triangle ABC touches the sides in X, Y and Z. In triangle XYZ the feet of the altitude from X and Y are X' and Y', respectively. Let line $X'Y'$ intersect the circumcircle of triangle ABC at P and Q. Prove that points X, Y, P and Q are concyclic. (Proposed by *László Simon,* Budapest) **A. 841.** Find all non-negative integer solutions of the equation $2^a + p^b = n^{p-1}$, where p is a prime number. (Proposed by *Máté Weisz,* Cambridge)

Problems in Physics

(see page 570)

M. 418. Measure the rotational inertia of a ball (e.g. football, table tennis ball or tennis ball) by rolling it down a slope. Give the result also in units of mR^2 (where R is the radius of the ball, m is its mass). Can the result be used to infer the thickness of the wall of the ball?

G. 797. An inflated balloon is attached to the open end of a liquid-column manometer. The difference in the level of petroleum in the two arms of the manometer is 72 cm. How many mm would the difference in level be if the manometer contained mercury? What is the gauge pressure (excess pressure) in the balloon? **G. 798.** In a 100 m flat race, the competitors will start from a kneeling start. The *figure* shows the horizontal force applied to the front and rear sensors in the starting machine when an athlete weighing 70 kg starts. Estimate the speed at which the athlete leaves the starting machine. **G. 799.** What is the minimum speed and maximum angle at which a body must be launched in order that it flies through a 100 metre long and 5 metre high straight tunnel? The air drag is negligible. **G. 800.** An object is located at a certain distance from