Nyíregyháza) **B. 5275.** Is there an irrational number a such that $a^{\sqrt{3}}$ is rational? (*5 points*) (Proposed by *B. Hujter,* Budapest) **B. 5276.** Prove that there exist infinitely many positive integers k for which the sum of the digits of 2^k is a) smaller; b) greater than the sum of the digits of 2^{k+1}. (*6 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5277.** The centre of the inscribed circle of triangle ABC is I. The midpoint of the circular arc BCA is F, and line FI intersects the circumscribed circle again at point M. Show that line CM passes through the external centre of similitude of the inscribed circle and the circumscribed circle. (*6 points*) (Proposed by *G. Kós,* Budapest)

New problems – competition A (see page 483): **A. 836.** For every $i \in \mathbb{N}$ let A_i, B_i and C_i be three finite and pairwise disjoint subsets of $\mathbb{N}$. Suppose that for every partition of $\mathbb{N}$ consisting of sets A, B and C there exists $i \in \mathbb{N}$ such that $A_i \subset A$, $B_i \subset B$ and $C_i \subset C$. Prove that there also exists a finite $S \subset \mathbb{N}$ such that for every partition of $\mathbb{N}$ consisting of sets A, B and C there exists $i \in S$ such that $A_i \subset A$, $B_i \subset B$ and $C_i \subset C$. (Submitted by *András Imolay,* Budapest) **A. 837.** Let all the edges of tetrahedron $A_1A_2A_3A_4$ be tangent to sphere S. Let a_i denote the length of the tangent from A_i to S. Prove that $\Big(\sum\limits_{i=1}^{4} \frac{1}{a_i}\Big)^2 > 2\Big(\sum\limits_{i=1}^{4} \frac{1}{a_i^2}\Big)$. (Submitted by *Viktor Vígh,* Szeged) **A. 838.** Sets $X \subset \mathbb{Z}^+$ and $Y \subset \mathbb{Z}^+$ are called comradely, if every positive integer n can be written as $n = xy$ for some $x \in X$ and $y \in Y$. Let $X(n)$ and $Y(n)$ denote the number of elements of X and Y, respectively, among the first n positive integers. Let $f : \mathbb{Z}^+ \to \mathbb{R}^+$ be an arbitrary function that goes to infinity. Prove that one can find comradely sets X and Y such that $\frac{X(n)}{n}$ and $\frac{Y(n)}{n}$ goes to 0, and for all $\varepsilon > 0$ exists $n \in \mathbb{Z}^+$ such that $\frac{\min\{X(n),Y(n)\}}{f(n)} < \varepsilon$.

Problems in Physics

(see page 506)

M. 417. Make a chain from 50 paper clips. Hold the chain vertically at one of its ends such that the other end just touches the table. Drop the chain several times in succession, and measure each time the maximum size of the tangled chain and the distance between the two ends of the chain. Calculate the mean and standard deviation of the measured values. Compare these with the total length of the chain when stretched.

G. 793. At a pressure of 1000 hPa a force equivalent to the weight of a 15 ton object is distributed over the body of a human being. a) what is the surface area of the body of the person? b) what is this weight at the highest point of the High Tatra Mountains? **G. 794.** The cross-section of a U-shaped tube is 1.5 cm^2. The tube is filled with mercury such that there is high enough mercury in both arms of the tube. 0.1 dl of water is poured on the mercury in one arm of the tube. In which arm and by what distance will the surface of the liquid be higher? **G. 795.** The angle between two plane mirrors is 60°. At a distance of 30 cm from the intersection of the two mirrors, a ray of light is incident on one of the mirrors, its angle of incidence is 30°. What is the minimum time that it takes for the reflected light to travel from one mirror to the other? **G. 796.** An ozone generator produces 5 g of ozone per hour by corona discharge and delivers it by a fan to the surface, which is to be disinfected. a) How many ozone molecules are produced in one hour? b) The manual recommends 30 minutes to disinfect a surface of area 28 m^2. The air is clean and dust-free, so the generated ozone will only decompose on the surface. Estimate the number of ozone molecules per bacterium on a surface area of 10 square microns.

P. 5436. Two cars, which can be considered to be point-like, are travelling at constant speed, each on a straight motorway towards the intersection of the motorways. (The angle between the two motorways is 90°.) The speed of car A is $v_A = 50$ km/h, the speed of car B is $v_B = 40$ km/h. At a given moment, the distance of the two cars from the intersection is $d_A = 20$ km, and $d_B = 36$ km. *a*) what will the minimum distance between the two cars be? *b*) How long will it be until they are closest? **P. 5437.** The mass of one of the starts of a binary star system is three times as much as that of the other. The two celestial bodies (whose size is much smaller than their distance) orbit in approximately circular orbits around their centre of mass. Which star has greater kinetic energy, with respect to the coordinate system fixed to the centre of mass of the system, and by what factor is it greater than that of the other star? **P. 5438.** On a Spanish farm, the olives are crushed to a pulp with the olive press shown in the figure. The plane of the crushing wheel, of diameter 90 cm, shown by the dashed line in the *figure,* is at a distance of 75 cm from the shaft. The wheel rolls without sliding. The tail of the donkey is 180 cm from the shaft, and the donkey undergoes circular motion at a speed of 2.4 m/s. An olive weighing 1 g got stuck to the crushing wheel. *a*) What is the speed of the olive when it is at the topmost point A of the wheel? *b*) what is the acceleration of the olive at this point? *c*) What is the magnitude and the direction of the force exerted by the wheel on the olive when it is at point A? **P. 5439.** A spherical copper ball at 20 °C hanging on a thin insulator string is immersed in a large amount of water at 80 °C. After a time of t_1 the copper ball warms up to 50 °C. Later the experiment is repeated in such a way that the initial temperature of water is 20 °C, while the ball is at 80 °C. In this way the copper ball cools down to 50 °C during a time of t_2. What is shorter, t_1 or t_2, if the ball *a*) is just immersed into the water, or *b*) is submerged almost to the bottom of the container? **P. 5440.** At a distance of 2 km from a beehive there is a black locust forest, from where a single bee can bring nectar of volume 30 mm^3 to the hive in each turn. 55% of the mass of the collected nectar is water, and when making honey, the worker bees in the hive make some portion of the water evaporate, such that the finished honey contains only 19% water. During the 12 days of flowering, the colony of the hive produces 25 kg of honey. The bees cover the energy requirements of evaporation by eating some part of the nectar they brought to the hive. *a*) How many watts is the average power output of the colony in the hive invested in the evaporation of the water? *b*) Altogether how many kilometres are covered by the worker bees while the total amount of nectar is carried to the hive? The density of nectar is $1.2\ \frac{\text{kg}}{\text{dm}^3}$; 1 kg nectar gives 6000 kJ energy; in order to evaporate 1 kg water the bees need 2400 kJ energy. **P. 5441.** We have formed a circle from a piece of metal wire, and from the same wire we would like to to make one of the chords between the two points of the circle. Where should the chord be in order that the equivalent resistance between the two end points of the chord is maximum, and what is the value of this largest equivalent resistance? Let R be the resistance of a wire which has a length equal to the radius of the circle. **P. 5442.** A nucleus, which was initially at rest, was accelerated though a potential difference of 20 kV, and then it enters into a uniform magnetic field of induction 1.0 T. The magnetic induction is perpendicular to the velocity of the nucleus. The magnetic field is separated from the force-free region by a plane perpendicular to the particle's direction of travel. The particle leaves the magnetic field after $3.3 \cdot 10^{-8}$ s. Which nucleus is it? **P. 5443.** KCl crystallises in a face-centred cubic system and has a lattice constant of 628 pm. What is the maximum wavelength of X-ray that can be used to create a Bragg reflection on the lattice planes perpendicular to the diagonal of the unit cell in the crystal? **P. 5444.** A small charged ball can move frictionlessly along a long, thin, vertical, insulating rod. If an equally small body of the same charge is placed at the bottom of the rod, the moving ball will be in equilibrium at a height of h_0. How far away from the rod in the horizontal direction can we move the lower body so that the ball on the rod can still be in equilibrium somewhere. What is the height of this is this position?