MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 72. No. 5. May 2022)

Problems in Mathematics

New exercises for practice – competition C (see page 289): **Exercises up to grade 10: C. 1721.** Bonnie listed 2022 numbers such that the ratio of the second number divided by the first number equals the third number on the list, and so on, for example, the seventh number equals the ratio of the sixth number divided by the fifth. What is the last number on Bonnie's list if the first number is 20, and the second number is 22? **C. 1722.** In a quadrilateral $ABCD$, sides AD and DC are equal in length. If α denotes the angle DAB then $\angle ABC = 2\alpha$, $\angle BCD = 3\alpha$ and $\angle CDA = 4\alpha$. Prove that side AB is twice as long as side AD. (*German competition problem*) **Exercises for everyone: C. 1723.** Determine all at most four-digit numbers $\overline{abcd}$ of distinct digits (allowing $a = 0$, too) for which $9 \cdot \overline{abcd} = \overline{acbcd}$. (Proposed by *A. Siposs,* Budapest) **C. 1724.** In a triangle ABC, $\angle CAB = 30^\circ$. Find the measures of the other angles of the triangle, given that the median drawn from vertex C encloses an angle of 45° with line AB. **C. 1725.** Let p denote a positive prime number. Given that the roots of the equation $x^2 - px - 580p = 0$ are integers, find the value of p. (Proposed by *M. Szalai,* Szeged) **Exercises upwards of grade 11: C. 1726.** Prove that if x, y, z are real numbers such that $\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 1$, then $\frac{x^2}{y+z} + \frac{y^2}{z+x} + \frac{z^2}{x+y} = 0$. Find all real numbers satisfying this condition. **C. 1727.** In a solid sphere of radius R, a cylindrical bore of radius $r < R$ is made along a line passing through its centre. Express the volume of the remaining solid in terms of the height m of the remaining solid. (Proposed by *B. Szabó,* Miskolc, 1986)

New exercises – competition B (see page 290): **B. 5246.** There are 14 people sitting around a table. Each of them is wearing either a blue shirt or a yellow shirt. What is the maximum possible number of people who have adjacent neighbors with shirts of different color? (*3 points*) **B. 5247.** The ends of a rope are fixed to the ground at two points separated by a distance shorter than the length of the rope. The rope will become taut if its midpoint is raised to a height of 150 cm. The rope will also become taut if a point of the rope 90 cm from one end is raised to a height of 90 cm. How long is the rope? (*3 points*) **B. 5248.** Solve the following simultaneous equations over the set of real numbers: $\frac{x^2}{y} + \frac{y^2}{x} + x + y = \frac{8}{xy}$, $x(x+1) + y(y+1) = 6$. (*4 points*) **B. 5249.** Let T_0 denote the area of the triangle formed by the points of tangency of the inscribed circle of triangle ABC on the sides, and let T_1 denote the area of the triangle formed by the centres of the escribed circles. Show that the geometric mean of T_0 and T_1 equals the area of triangle ABC. (*5 points*) (Proposed by *P. Bártfai*) **B. 5250.** Prove that for all non-negative integers n, $2^{2^n(n-2)+n+2} \leqslant (2^n)! \leqslant 2^{2^n(n-1)+1}$. (*5 points*) (Proposed by *I. Blahota,* Nyíregyháza) **B. 5251.** The vertices of a rectangle $ABCD$ in the coordinate plane are $A(0,0)$, $B(2022,0)$, $C(2022,2)$, $D(0,2)$. Consider those triangles of unit area that have all three vertices at lattice points lying on the longer sides of the rectangle. These triangles are to be coloured so that no triangles of the same colour have an interior point in common. What is the minimum number of colours needed? (*5 points*) (Proposed by *Z. L. Nagy* Budapest) **B. 5252.** A polyhedron $ABCA_1B_1C_1$ has six vertices. Faces ABC and $A_1B_1C_1$ are triangles. The edges AA_1, BB_1 and CC_1 are parallel. Faces AA_1B_1B, BB_1C_1C and CC_1A_1A are trapeziums in which the diagonals intersect at points P, Q and R, respectively. Show that the volumes of polyhedra $ABCPQR$ and $A_1B_1C_1PQR$ are equal. (*6 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5253.** Is it true that if $\binom{n}{k}$

is even, then the k-element subsets of an n-element set S can be paired up so that the symmetric difference of every pair should have exactly 2 elements? (*6 points*)

New problems – competition A (see page 291): **A. 827.** Let $n>1$ be a given integer. In a deck of cards the cards are of n different suites and n different values, and for each pair of a suite and a value there is exactly one such card. We shuffle the deck and distribute the cards among n players giving each player n cards. The players' goal is to choose a way to sit down around a round table so that they will be able to do the following: the first player puts down an arbitrary card, and then each consecutive player puts down a card that has a different suite and different value compared to the previous card that was put down on the table. For which n is it possible that the cards were distributed in such a way that the players cannot achieve their goal? (The players work together, and they can see each other's cards.) (Proposed by *Anett Kocsis,* Budapest) **A. 828.** Triangle ABC has incenter I and excircles Ω_A, Ω_B, and Ω_C. Let ℓ_A be the line through the feet of the tangents from I to Ω_A, and define lines ℓ_B and ℓ_C similarly. Prove that the orthocenter of the triangle formed by lines ℓ_A, ℓ_B, and ℓ_C coincides with the Nagel point of triangle ABC. (The Nagel point of triangle ABC is the intersection of segments AT_A, BT_B, and CT_C, where T_A is the tangency point of Ω_A with side BC, and points T_B and T_C are defined similarly.) (Proposed by *Nikolai Beluhov,* Bulgaria) **A. 829.** Let G be a simple graph on n vertices with at least one edge, and let us consider those $S:\,V(G)\to\mathbb{R}^{\geqslant 0}$ weighings of the vertices of the graph for which $\sum\limits_{v\in V(G)} S(v)=1$. Furthermore define $f(G)=\max\limits_{S}\min\limits_{(v,w)\in E(G)} S(v)S(w)$, where S runs through all possible weighings. Prove that $f(G)=\frac{1}{n^2}$ if and only if the vertices of G can be covered with a disjoint union of edges and odd cycles. ($V(G)$ denotes the vertices of graph G, $E(G)$ denotes the edges of graph G.)

Problems in Physics

(see page 313)

M. 414. Measure the coefficient of kinetic friction between several sheets of sandpaper with different grit sizes and a wooden block.

G. 781. Boil water in a large pot on the stove. Put some cool water in a thin-walled glass, then immerse the glass of water into the boiling water so that it does not touch the walls of the pot. Will the water in the glass boil if we wait for a long enough time? **G. 782.** A bicycle is moving uniformly along a horizontal path at a speed of 3 m/s. Its wheels have a diameter of 70 cm. Choose an arbitrary point on the circumference of the wheel and at different positions of the wheel draw the velocity vectors and the acceleration vectors of this point starting from one common point for each quantity, that is draw the velocity and acceleration hodographs. **G. 783.** There is a point-like light source at the centre of a uniform glass ball of radius R and of refractive index n. The sphere is observed from the outside. Where do we see the image of the light source? **G. 784.** The *figure* shows a whole range of simple machines. Friction and the masses of pulleys and levels are negligible. Into which direction will the lowermost object start moving?

P. 5409. The *figure* shows a whole range of simple machines. Friction and the masses of pulleys and levels are negligible. What are the values of the tension in the threads? **P. 5410.** The peregrine falcon can travel long distances without flapping its wing. Doing so, its movement has two parts. In the first part, it circles with its wings extended and rises in an upward flowing column of warm air (thermals) at a vertical speed of v_1. In