Áprilisi pótfeladat.* A KöMaL minden számát a nyomdába adás előtt ketten is elolvassák. A mostani számban az egyik lektor 60, a másik 40 hibát talált, és ezek között 35 volt olyan, amelyet mindketten észrevettek. Becsüljük meg, hogy hány hiba maradhatott ezek után a kéziratban!

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Beküldési határidő: 2022. május 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 72. No. 4. April 2022)

Problems in Mathematics

New exercises for practice – competition C (see page 226): **Exercises up to grade 10: C. 1714.** The integers 1 to 22 are written on a blackboard. In each move, a pair of numbers is selected, erased and replaced with the absolute value of their difference. Prove that the last number added to the board is odd. (*German problem*) **C. 1715.** A circle k_1 of radius 8 cm lies in the interior of a circle k. Both circles intersect the circle k_2 of radius 15 cm, as shown in the *figure.* What is the radius of k if the shaded area inside k but outside k_1 is equal to the total area of the shaded regions in the interior of k_2? **Exercises for everyone: C. 1716.** In factorial representation, the place values of the digits are not the powers of a base: the nth place value is n factorial. Thus the digit in the first place is to be multiplied by 1, the digit in the second place is multiplied by 2, that in the third place is multiplied by 6, and so on. For example, the number $3310_!$ in factorial representation corresponds to the number $3\cdot 4!+3\cdot 3!+1\cdot 2!=92$ in decimal notation. (If there is a number of more than one decimal digits in a certain place then it is indicated by using brackets.)† It is observed that one third of $111_!$ is $11_!$, one third of $111\,111_!$ is $22\,011_!$, and one third of $111\,111\,111_!$ is $33\,022\,011_!$. Determine the factorial representation of one third of the number that consists of $3n$ digits of 1, also given in factorial representation. (Based on the idea of *I. Lénárt,* Budapest) **C. 1717.** Let x_1 and x_2 denote the two real roots of the equation $15x^2-21x+7=0$. Find the exact value of the expression $\frac{x_1}{x_2}+\frac{x_2}{x_1}+\frac{1}{x_1}+\frac{1}{x_2}$. **C. 1718.** Eight unit cubes are placed on a plane, and five further unit cubes are placed on top of them as shown in the *figure.* Find the lengths of the sides of triangle ABC. **Exercises upwards of grade 11: C. 1719.** In the interior of a regular triangle ABC consider the points P where side AB subtends an angle of 135°. Prove that the line segments PA, PB, PC can always form a triangle, and one angle of such triangles is always the same, independently of the position of point P. **C. 1720.** The elements of a given set of 10 elements are all at most two-digit positive integers. Is it true that such a set will always have two disjoint subsets in which the sum of the elements is equal?

* A pótfeladat megoldása beküldhető a `szerk@komal.hu` címre, de nem számít bele a pontversenybe.

† It can be shown that the representation is unique, that is, every positive integer has a single factorial representation. See the Informatics problems **I. 553.** of the January issue.

New exercises – competition B (see page 227): **B. 5238.** Solve the following equation over the set of positive integers: $(k+n)! = k^3 + n^3 + (k+n)(3kn-1)$. (*3 points*) (Proposed by *M. Szalai,* Szeged) **B. 5239.** The sides a, b and c of a triangle, in this order, form an arithmetic sequence. Show that the centre of the inscribed circle divides the angle bisector drawn to side b in a $1:2$ ratio. (*3 points*) **B. 5240.** Show that every positive integer n has a multiple in which the sum of the digits is n. (*4 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5241.** In a triangle ABC, the centre of the circumscribed circle is O, and $\angle ABC > 90^\circ$. The tangent drawn to the circumscribed circle at C intersects line AB at point P, and the perpendicular drawn from P to BC intersects line OC at Q. Prove that AB is perpendicular to AQ. (*4 points*) (Proposed by *Z. L. Nagy,* Budapest) **B. 5242.** Let m and n denote arbitrary positive integers. Consider those lattice points $(x;y)$ in the Cartesian coordinate plane for which $1 \leqslant x \leqslant m$ and $1 \leqslant y \leqslant n$. What is the maximum possible number of points that can be selected out of these mn lattice points such that no four points selected should form a non-degenerate parallelogram? (*6 points*) (Proposed by *E. Füredi,* Budapest) **B. 5243.** In a triangle ABC, $\angle CAB = 48^\circ$ and $\angle ABC = 54^\circ$. D is an interior point of the triangle such that $\angle CDB = 132^\circ$ and $\angle BCD = 30^\circ$. Prove that the line segments forming the polygon $ACDB$ cannot form a triangle. (*5 points*) **B. 5244.** Determine all integers $n > 4$ such that $(k,n) > 1$ for all composite numbers k less than n. (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5245.** *a*) Prove that there exist infinitely many, pairwise non-similar triangles in which the lengths of the sides are integers, and one angle is 3 times as large as another. *b*) Is there a triangle with the above property in which the lengths of the sides are all at most 10? (*6 points*) (Based on the idea of *M. Hujter,* Budapest) **April puzzle*****.** Place on a chessboard six white chess pieces selected from two chess sets such that however a black piece is placed on a vacant field, it can be eliminated immediately.

New problems – competition A (see page 229): **A. 824.** An infinite set S of positive numbers is called thick, if in every interval of the form $\big[1/(n+1), 1/n\big]$ (where n is an arbitrary positive integer) there is a number which is the difference of two elements from S. Does there exist a thick set such that the sum of its elements is finite? (Proposed by *Gábor Szűcs,* Szikszó) **A. 825.** Find all functions $f:\mathbb{Z}^+ \to \mathbb{R}^+$ that satisfy $f(nk^2) = f(n)f^2(k)$ for all positive integers n and k, furthermore $\lim\limits_{n\to\infty} \dfrac{f(n+1)}{f(n)} = 1$. **A. 826.** An antelope is a chess piece which moves similarly to the knight: two cells (x_1, y_1) and (x_2, y_2) are joined by an antelope move if and only if $\big\{|x_1 - x_2|, |y_1 - y_2|\big\} = \{3, 4\}$. The numbers from 1 to 10^{12} are placed in the cells of a $10^6 \times 10^6$ grid. Let D be the set of all absolute differences of the form $|a-b|$, where a and b are joined by an antelope move in the arrangement. How many arrangements are there such that D contains exactly four elements? (Proposed by *Nikolai Beluhov,* Bulgaria)

Problems in Physics

(see page 251)

M. 413. Measure the refractive index of cooking oil.

G. 777. We would like to heat room temperature espresso coffee by "steaming". Estimate how much the "quality" of the coffee deteriorates, that is, how much the coffee concentration decreases. **G. 778.** The *figure* shows the wet track of a bicycle on a dry asphalt after the bicycle passed a puddle. Did the bike move from the left to the right or

* Out of competition. A possible solution will be shown on the cover of the May issue.