and T_c, respectively. *a*) Prove that lines $A'A_a$, $B'B_b$ and $C'C_c$ are concurrent. *b*) Prove that lines $A'T_a$ and $B'T_b$ and $C'T_c$ are also concurrent, and their point of intersection is on the line defined by the orthocentre and the incentre of triangle ABC. (Submitted by *Viktor Csaplár,* Bátorkeszi and *Dániel Hegedűs,* Gyöngyös)

Problems in Physics

(see page 122)

M. 411. Measure the rotational inertia of an empty beer bottle about its symmetry axis. In order to make the measurement more accurate, carry out the measurement in two different ways. Compare the accuracy of the two measurements.

G. 769. The fuel consumption meter of a car, moving uniformly along a level road, reads 5 litres/100 km. If the same car is moving along the same road, and accelerates uniformly, then the reading on the fuel consumption meter at the moment when the car reaches the speed of the uniform motion is 10 litres/100 km. When the car goes at the same speed on a hill the fuel consumption is 12 litres/100 km. What does the meter read when the car is driving up the same hill at the acceleration as described before at the moment when its speed also reaches the value described above? **G. 770.** The most common heating fuel is natural gas. The gas utility company uses the following method to determine the monthly price to be paid, which is shown on the gas bill (in Hungarian): the amount of consumed gas is multiplied by a so-called correction factor, then the heating value of the gas in MJ is calculated, and finally the heating value is used to calculate the amount of money to be paid. To this, a basic monthly household charge is added. Find a recent gas bill and a recent electricity bill and answer the following questions (in your answers always give the gross values!) *a*) Why is it necessary to use the correction factor? *b*) What is the price of 1 m^3 natural gas at standard conditions? *c*) How much does 1 MJ of energy cost if it comes into your home as gas or as electricity? **G. 771.** In the connection shown in the *figure,* the resistors have the same resistance R, and at a voltage U the their power is P. What is the power dissipated in each resistor with the switches open (O) or closed (C)? Fill in the table!

1.	2.	A	B	C
O	O			
O	C			
C	O			
C	C			

G. 772. Children play a circle game in the field. Unfortunately, the child in the middle of the circle steps in a wasps' nest and the angry wasps fly away. The wind is blowing from the east at a speed of 4.5 m/s in the field, and the children are running radially outward at a speed of 6 m/s. According to the measurements of scientists these wasps in calm conditions can fly at a speed of 8 m/s. Estimate the percentage of children who are surely safe from wasp stings! You can also use a ruler, a pair of compasses and a protractor to find out the answer.

P. 5382. An old tape magnetophone spins the takeup reel at a constant speed during a fast rewind. The inner diameter of both reels is 5 cm and their outer diameter is 15 cm. From the fully loaded feed reel, the rewinding time of the tape is 3 minutes. The tape is wound onto the initially empty takeup reel. How much time elapses from the start until the two reels have equal lengths of tape wound on them? **P. 5383.** What would the mass of the Earth have to be — with unchanged rotation and diameter — so that we would not be able to receive the satellite TV broadcast with a parabolic dish in Budapest. **P. 5384.** There is a small ball placed on top of a thin, uniform, vertical stick. Compared to the mass of the stick, the mass of the ball is negligible and the stick stands on a table which can be considered frictionless. Suddenly the stick falls over. In which case will the ball strike the tabletop at a higher speed, if it is glued to the top of the stick, or if it is simply put on the stick, from where it can fall off very easily? **P. 5385.** To what fraction does the heat flux released through a window, which has a single glass layer in it, decreases if the window is replaced by a double pane one? The thickness of each glass layer in both cases is $d_{\mathrm{glass}} = 3$ mm, and in the case of the double pane window there is a $d_{\mathrm{air}} = 7$ mm air gap between the two glass layers. The thermal conductivity of air is $\kappa_{\mathrm{air}} = 0.025$ W/(m K) and the thermal conductivity of glass is $\kappa_{\mathrm{glass}} = 1.2$ W/(m K). **P. 5386.** There is a small ball of charge $Q = 5.55$ μC fixed at the bottom of a 2 m long trough made of some insulating material. The trough makes an angle of elevation of $\alpha = 30^\circ$ with the horizontal. From the top of the trough another small ball of mass $m = 100$ g with charge $q = 10$ μC is released from rest. How far does this ball can move if it rolls without slipping? (The charge of the ball does not change during its motion.) **P. 5387.** Different resistors of resistances R are connected to a battery of electromotive force U_0 and of internal resistance R_{b}. *a*) What is the maximum "useful" power (dissipated in the external resistor) that this battery can deliver? At what external resistance value R can we achieve this maximum power $P_{\max}$? *b*) Show that for any other power P which is smaller than $P_{\max}$, there are two external resistors of resistances $R_1 \neq R_2$ at which the dissipated power is P. What is the arithmetic and the geometric mean of R_1 and R_2? *c*) What is the sum of the terminal voltages across the battery in the above two cases? *d*) What is the sum of the currents through R_1 and R_2? *e*) The efficiency of the delivered energy is defined as the ratio of the useful power to the total power delivered by the battery. What is the sum of the efficiencies in the above two cases? **P. 5388.** Linearly polarized light from a 15 mW laser having a wavelength of $\lambda = 632.8$ nm is emitted from the 2 mm diameter circular aperture of the laser box. *a*) What is the maximum value of the electric field in the laser beam? *b*) What is the total linear momentum of a one metre long piece of the laser beam? **P. 5389.** A (point-like) fly flies at a constant speed of v parallel to the principal axis of a lens, having a focal length of f, at a distance of d from it. What is the least speed of the fly with respect to its image? **P. 5390.** There is a small electric dipole of dipole moment p at the centre of the thin-walled uncharged metal spherical shell of radius R, shown in the *figure*. Determine the surface charge density at points A and B, which are interior points of the shell. Determine the surface density of the charge on the outer surface of the shell as well. (*Hint*: Use the method of image charges applied for a sphere. It might be also useful to know the electric field due to a dipole at a point on the axial and equatorial lines.)

Problems of the 2021 Kürschák competition

1. In the Cartesian coordinate system of the plane, the triangle determined by the points $P_i = (a_i, b_i)$ $(i = 0, 1, 2)$ contains the origin $O = (0, 0)$ in its interior. Show that the areas of the triangles P_0OP_1, P_0OP_2, P_1OP_2 (in this order) form a geometric sequence if and only if the system of equations $a_0x^2 + a_1x + a_2 = b_0x^2 + b_1x + b_2 = 0$ has a real solution x.

2. In Wonderland, n airlines operate flights between n cities. For each airline, there are an odd number of cities, say, $v_1, v_2, \dots, v_i$ such that the airline operates the following flights: v_jv_{j+1} and $v_{j+1}v_j$ for $1 \leqslant j \leqslant i$, where $v_{i+1} = v_i$. Prove that we may choose an odd number of cities, say, $u_1, u_2, \dots, u_k$ in such a way that it is possible to buy tickets for the flights $u_1u_2, u_2u_3, \dots, u_{k-1}u_k, u_ku_1$ from pairwise different airlines.

3. In the cyclic hexagon $A_1B_3A_2B_1A_3B_2$, the diagonals A_1B_1, A_2B_2 and A_3B_3 are concurrent. For $i = 1, 2, 3$, let C_i be the intersection of the diagonals A_iB_i and $A_{i+1}A_{i+2}$, and let D_i be a point on the circumscribed circle, different from B_i, such that the circle $B_iC_iD_i$ is tangent to the line $A_{i+1}A_{i+2}$. (The points are indexed modulo 3, that is, $A_4 = A_1$ and $A_5 = A_2$.) Prove that the segments A_1D_1, A_2D_2 and A_3D_3 are concurrent.