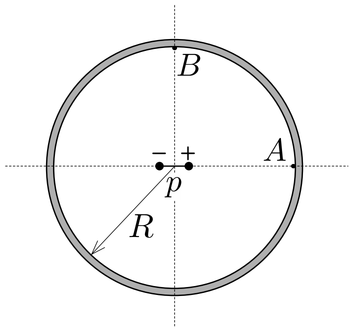
P. 5389. Egy (pontszerűnek tekinthető) légy repül állandó v sebességgel az f fókusztávolságú lencse optikai tengelyével párhuzamosan, attól d távolságra. Legalább mekkora nagyságú a légy és a légy képének relatív sebessége?

(*5 pont*) *Észtországi versenyfeladat* nyomán

P. 5390. Az *ábrán* látható R sugarú, vékony falú, töltetlen fémgömbhéj középpontjában egy kicsiny, p dipólmomentumú elektromos dipólus helyezkedik el. Határozzuk meg a gömbhéj belső felületén lévő A és B pontokban a felületi töltéssűrűséget! Adjuk meg a fémgömbhéj külső felületén lévő töltéssűrűséget is!

(*Útmutatás*: Alkalmazhatjuk a gömbi tükörtöltés módszerét. Hasznos lehet még a dipólus elektromos terének ismerete az ún. Gauss-féle főhelyzetekben.)



(*6 pont*) Közli: *Szász Krisztián,* Budapest

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Beküldési határidő: 2022. március 15.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`

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Problems in Mathematics

New exercises for practice – competition K (see page 89): **K. 719.** Each integer on the number line is coloured either red or blue. Is it certain for all possible colourings that a) there will be two numbers of the same colour separated by a distance of 3; b) there will be two numbers of the same colour separated by a distance of 3 or 4? **K. 720.** Divide the area of a regular hexagon into three equal parts with two lines passing through the same vertex. **K. 721.** Alex made some wooden sticks of integer lengths such that no three of them could be used to form a triangle. Given that there were sticks of lengths 1 and 10 and that the longest stick was 100 units long, what is the maximum possible number of sticks that Alex may have made? **K/C. 722.** The arithmetic mean of two three-digit numbers equals the number obtained by writing them next to each other, separated by a decimal point. What may be the two numbers? **K/C. 723.** The Hungarian Handball Federation nominated 17 players for women's handball in the Tokyo Olympic Games: 3 goalkeepers, 1 right winger, 4 right backs, 2 playmakers, 3 pivots, 2 left backs and 2 left wingers. In how many different ways may the players line up for the anthem if players of the same position must stand together? (During the anthem, players line up next to each other in a single line.) (Proposed by *B. Róka* Budapest)

New exercises for practice – competition C (see page 90): **Exercises up to grade 10: K/C. 722.** See the text at Exercises **K. K/C. 723.** See the text at Exercises **K. Exercises**

for everyone: C. 1704. For which real numbers a will the minimum of the function $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$ defined on the segment $[0;2]$ be equal to 3? (*MC&IC*) **C. 1705.** Given that a certain quadrilateral is a kite, it is cyclic and its sides are 42 and 56 units long, what is the distance between the centres of the inscribed and circumscribed circles? (Proposed by *A. Siposs,* Budapest) **C. 1706.** Prove that in every set of 2022 positive integers there exist two numbers such that their difference or sum is divisible by 4040. (Proposed by *L. Sáfár,* Ráckeve) **Exercises upwards of grade 11: C. 1707.** In a triangle ABC (with the usual notations) $b=6$, $a=2$ and they enclose an angle of $\gamma = 120^\circ$. Find the exact length of the interior angle bisector of angle γ. (*MC&IC*) **C. 1708.** Solve the following equation over the set of real number pairs: $\log_2^2(x+y) + \log_2^2(xy) + 1 = 2\log_2(x+y)$. (*MC&IC*)

New exercises – **competition B** (see page 91): **B. 5222.** Let A denote the set of even positive integers for which the sum of the digits decreases by 2 if the number is halved. Let B denote the set of positive integers for which the sum of the digits increases by 5 if the number is multiplied by 5. What is the number of elements in the set $A \cap B$ and in the set $B \setminus A$? (*3 points*) (Proposed by *T. Káspári,* Paks) **B. 5223.** Define the sequence $\{a_n\}$ as follows: $a_1 = -3$, $a_{n+1} = 4 + a_n + 4\sqrt{a_n + 4}$. Determine the value of a_{2022}. (*3 points*) (Proposed by *T. Káspári,* Paks) **B. 5224.** P is a point on side BC of a unit square $ABCD$, and Q is a point on side CD, such that $\angle PAQ = 45^\circ$. For which positions of points P and Q will the sum $BP + PQ + QD$ be minimal? (*4 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5225.** The inscribed circle of triangle ABC is centred at I and has a radius ϱ, the radius of the circumscribed circle is R. Prove that if $\overline{AI} = R$, then the area of the triangle ABC is $\frac{a\cdot R}{4} + \varrho \cdot a$, where a denotes the length of the side opposite to vertex A. (*4 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5226.** The length of each side of a triangle is at most 2 units. Each pair of vertices is joined with an arc of a unit circle, not longer than a semicircle. Prove that $a' + b' > 2c'/3$, where a', b', c' denote the lengths of the arcs. (*5 points*) **B. 5227.** Give an example of a positive integer k, along with a finite tree graph F of at least k vertices in which the degree of each vertex is at most 3, and F will fall apart into at least 2022 components if an arbitrary connected subgraph of k vertices is deleted from F. (*6 points*) (Based on a *Monthly problem*) **B. 5228.** A parabola intersects side AB of a triangle ABC at interior points C_1 and C_2, side BC at interior points A_1 and A_2, and it intersects side CA at interior points B_1 and B_2. Prove that if $AC_1 = C_2B$ and $BA_1 = A_2C$ then $CB_1 = B_2A$. (*5 points*) (Proposed by *G. Holló,* Budapest) **B. 5229.** Let $a \neq 0$ be a real number, and let $f : \mathbb{R} \to \mathbb{R}$ be a function, such that $f\big(x + f(y)\big) = f(x) + f(y) + ay$ for all $x, y \in \mathbb{R}$. Prove that f is additive, that is, $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. (*6 points*) (Proposed by *G. Stoica,* Saint John, New Brunswick, Canada)

New problems – **competition A** (see page 92): **A. 818.** Find all pairs of positive integers m, n such that $9^{|m-n|} + 3^{|m-n|} + 1$ is divisible by m and n simultaneously. (Submitted by *Géza Kós,* Budapest) **A. 819.** Let G be an arbitrarily chosen finite simple graph. We write non-negative integers on the vertices of the graph such that for each vertex v in G the number written on v is equal to the number of vertices adjacent to v where an even number is written. Prove that the number of ways to achieve this is a power of 2. **A. 820.** Let ABC be an arbitrary triangle. Let the excircle tangent to side a be tangent to lines AB, BC and CA at points C_a, A_a and B_a, respectively. Similarly, let the excircle tangent to side b be tangent to lines AB, BC and CA at points B_c, B_a and B_b, respectively. Finally, let the excircle tangent to side c be tangent to lines AB, BC and CA at points C_c, A_c and B_c, respectively. Let A' be the intersection of lines A_bC_b and A_cB_c. Similarly, let B' be the intersection of lines B_aC_a and A_cB_c, and let C' be the intersection of lines B_aC_a and A_bC_b. Finally, let the incircle be tangent to sides a, b and c at points T_a, T_b

and T_c, respectively. *a*) Prove that lines $A'A_a$, $B'B_b$ and $C'C_c$ are concurrent. *b*) Prove that lines $A'T_a$ and $B'T_b$ and $C'T_c$ are also concurrent, and their point of intersection is on the line defined by the orthocentre and the incentre of triangle ABC. (Submitted by *Viktor Csaplár,* Bátorkeszi and *Dániel Hegedűs,* Gyöngyös)

Problems in Physics

(see page 122)

M. 411. Measure the rotational inertia of an empty beer bottle about its symmetry axis. In order to make the measurement more accurate, carry out the measurement in two different ways. Compare the accuracy of the two measurements.

G. 769. The fuel consumption meter of a car, moving uniformly along a level road, reads 5 litres/100 km. If the same car is moving along the same road, and accelerates uniformly, then the reading on the fuel consumption meter at the moment when the car reaches the speed of the uniform motion is 10 litres/100 km. When the car goes at the same speed on a hill the fuel consumption is 12 litres/100 km. What does the meter read when the car is driving up the same hill at the acceleration as described before at the moment when its speed also reaches the value described above? **G. 770.** The most common heating fuel is natural gas. The gas utility company uses the following method to determine the monthly price to be paid, which is shown on the gas bill (in Hungarian): the amount of consumed gas is multiplied by a so-called correction factor, then the heating value of the gas in MJ is calculated, and finally the heating value is used to calculate the amount of money to be paid. To this, a basic monthly household charge is added. Find a recent gas bill and a recent electricity bill and answer the following questions (in your answers always give the gross values!) *a*) Why is it necessary to use the correction factor? *b*) What is the price of 1 m^3 natural gas at standard conditions? *c*) How much does 1 MJ of energy cost if it comes into your home as gas or as electricity? **G. 771.** In the connection shown in the *figure,* the resistors have the same resistance R, and at a voltage U the their power is P. What is the power dissipated in each resistor with the switches open (O) or closed (C)? Fill in the table!

1.	2.	A	B	C
O	O			
O	C			
C	O			
C	C			

G. 772. Children play a circle game in the field. Unfortunately, the child in the middle of the circle steps in a wasps' nest and the angry wasps fly away. The wind is blowing from the east at a speed of 4.5 m/s in the field, and the children are running radially outward at a speed of 6 m/s. According to the measurements of scientists these wasps in calm conditions can fly at a speed of 8 m/s. Estimate the percentage of children who are surely safe from wasp stings! You can also use a ruler, a pair of compasses and a protractor to find out the answer.

P. 5382. An old tape magnetophone spins the takeup reel at a constant speed during a fast rewind. The inner diameter of both reels is 5 cm and their outer diameter is 15 cm. From the fully loaded feed reel, the rewinding time of the tape is 3 minutes. The tape is wound onto the initially empty takeup reel. How much time elapses from the start until the two reels have equal lengths of tape wound on them? **P. 5383.** What would the mass of the Earth have to be — with unchanged rotation and diameter — so that we would not be able to receive the satellite TV broadcast with a parabolic dish in Budapest. **P. 5384.** There is a small ball placed on top of a thin, uniform, vertical stick. Compared to the mass of the stick, the mass of the ball is negligible and the stick stands on a table which can be considered frictionless. Suddenly the stick falls over. In which case will the ball strike the tabletop at a higher speed, if it is glued to the top of the stick, or if it is simply