P. 5363. Egy vékony, magas üvegcsőből homokórát készítettünk. A benne lévő homok m_0 tömege megegyezik az üvegcső és a tartótalpak együttes tömegével. Kezdetben a homok az alsó térfél $h=5$ cm hosszú részét tölti ki, és az eszköz megfordítása után egyenletes ütemben $t_0 =$ $= 1$ perc alatt pereg le. (A felső és az alsó térfélben lévő homok alakját közelítsük hengerekkel.)

a) Határozzuk meg, hogy hol van a homokóra tömegközéppontja t idővel az óra elindítása után! (Ne foglalkozzunk a homokóra indítását követő, illetve a megállását közvetlenül megelőző nagyon rövid időtartamokkal, amikor a homokzuhatag még vagy már nem tölti ki a kifolyónyílás és az alsó becsapódási hely közötti teljes távolságot.)

b) Számítsuk ki, hogy mekkora a homokóra impulzusa (lendülete) t idővel a homokóra elindítása után!

c) Nagyon érzékeny mérleggel megmérjük a homokóra súlyát, miközben a homok a felső tartályból az alsóba pereg. Azt találjuk, hogy a mért súly egy kicsivel nagyobb, mint a már lepergett homokóra súlya. Az előző két részfeladatra adott választ felhasználva adjuk meg, hogy hány ezrelékkel nagyobb a működő homokóra súlya a már „lejárt” homokóráénál!

(*6 pont*) Közli: *Széchenyi Gábor,* Budapest

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Problems in Mathematics

New exercises for practice – competition K (see page 477): **K. 704.** There were 5 participants in a chess tournament. Each player played every other player once. 1 point was awarded for winning the game, 0.5 point for a draw and 0 for losing. At the end, it turned out that: – the player finishing in the first place had no draws; – the player in the second place lost no game; – each player had a different number of points. Find the score of each player. **K. 705.** Three different numbers are chosen from 1, 2, 3, 4, 5, 6, 7, 8 and added. This is performed for every possible selection of three numbers. Some of the sums obtained will be even and some will be odd. Which kind of result will occur more frequently: even or odd? **K. 706.** Three numbers a, b, c are entered (left to right) in the first row of a three column table. The numbers in the second row are $a-b$, $b-c$, $c-a$. In the third row, the numbers are obtained by the same rule from the second row (the same operations carried out with the numbers of the first, second and third fields), and so on. Show that from the fourth row onwards 2021 cannot occur in the table. **K/C. 707.** A few children (at least two) are standing around a circle. They are playing an "elimination game" as follows: counting from the starting player, every second child is eliminated from the circle. The player remaining in the circle alone will win the game. For example, if there are six players A, B, C, D, E, F and A starts then the players eliminated (in this order) are B, D, F, C, A. Thus the winner is E. With how many players can the starting player win the game? **K/C. 708.** Jean the butler is instructed by his lord to place candles in the 10 three-prong candle holders in the living room. Jean either needs to put three candles of different colours in each holder, or put 30 candles of the same colour in all of them. Jean goes to the shop on the corner to buy the candles, and finds that the shop only has 70 candles altogether. Show that Jean can buy an appropriate selection of 30 candles.

New exercises for practice – competition C (see page 478): **Exercises up to grade 10: K/C. 707.** See the text at Exercises **K. K/C. 708.** See the text at Exercises **K. Exercises for everyone: C. 1689.** Solve the following simultaneous equations for integers a, b, c, d: $a+d=9$, $ad+b=8$, $bd+c=74$, $cd=18$. (Proposed by *E. Berkó,* Szolnok) **C. 1690.** The centre of a semicircle of unit radius and diameter AB is O. The semicircle of diameter OB, centred at K is drawn inside the larger semicircle. A ray drawn from point A touches the small semicircle at point C. The perpendicular dropped from point O to AC intersects the arc of diameter AB at a point D. Prove that the midpoint of line segment BD is C. **C. 1691.** What are the positive prime numbers p, q for which $p^5-q^3+(p+q)^4=9900$? **Exercises upwards of grade 11: C. 1692.** P is an interior point of side DA of a square $ABCD$. The angle bisector of $\angle PBC$ intersects side CD at point Q, and the foot of the perpendicular dropped from point Q to line BP is R. Find the angle of the lines AR and BQ. **C. 1693.** Four vertices of a cube are selected at random. Every selection of four vertices is equally probable. What is the probability that the four vertices form a tetrahedron? What is the probability that the four vertices form a regular tetrahedron? (Proposed by *N. Zagyva,* Baja)

New exercises – competition B (see page 479): **B. 5198.** I have a tortoise, a cat and a dog. If the cat is standing on the floor and I place the tortoise on the table, the head of the tortoise will be 70 cm above the head of the cat. If the dog is standing on the floor and I place the cat on the table, the head of the cat will be 80 cm above the head of the dog. Finally, if the tortoise is standing on the floor and I place the dog on the table, the head of the dog will be 120 cm above the head of the tortoise. How tall is the table? (*3 points*) (Based on the idea of *Sz. Kocsis,* Budapest) **B. 5199.** A coin is placed on each field of a chessboard, heads facing up. In each move, we can (simultaneously) turn over three adjacent coins in any row or column. Is it possible to achieve an arrangement where all coins show tails on top? (*4 points*) (Proposed by *M. E. Gáspár,* Budapest) **B. 5200.** The diameter of a semicircular arc is $A_0A_1=1$. A point A_2 is selected on the arc, such that $\angle A_0A_1A_2=1^\circ$. Then a point A_3 is selected on the arc A_1A_2, such that $\angle A_1A_2A_3=2^\circ$. The procedure is continued: point A_{k+1} is selected on the arc $A_{k-1}A_k$, so that the measure of angle $A_{k-1}A_kA_{k+1}$ is k degrees $(k=3,4,\dots,9)$. What will be the length of the line segment A_9A_{10}? (The *figure* is not to scale.) (*3 points*) **B. 5201.** Let $1=d_1<d_2<\dots<d_k=n$ be the divisors of a positive integer n. Determine those composite numbers n for which the numbers d_1, d_1+d_2, $d_1+d_2+d_3$, $\dots$, $d_1+d_2+\dots+d_{k-1}$ all divide n. (*4 points*) (Proposed by *Cs. Sándor,* Budapest) **B. 5202.** Two rational numbers are said to be *acquainted to each other* if they can be represented in the forms p/q and r/s, respectively (p, q, r, s are integers), such that $|ps-qr|=1$. If two rational numbers are acquainted to each other, how many common acquaintances may they have? (*5 points*) (Proposed by *Sz. Kocsis,* Budapest) **B. 5203.** In a triangle ABC, $AB>BC$, the inscribed circle touches sides BC, CA and AB at the points A_0, B_0 és C_0, respectively, and the escribed circle drawn to side AC touches AC at point B_1. Show that the intersection of the line segments A_0B_1 and B_0C_0 lies on the interior angle bisector drawn from vertex B if and only if the angle at vertex C measures 90°. (*5 points*) (Proposed by *G. Holló,* Budapest) **B. 5204.** Let $1\leqslant a,b,c,d\leqslant 4$ denote real numbers. Prove that $16\leqslant (a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)\leqslant 25$. (*6 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5205.** There are four circles given in the plane: circle k_2 lies in the interior of circle k_1, circle k_3 lies in the interior of k_2, and circle k_4 lies in the interior of k_3. Also given are three lines e_1, e_2 and e_3 which are pairwise non-parallel and each line intersects each circle. For all $i = 1, 2, 3$ let the intersections of line e_i with the circles be A_i, B_i, C_i, D_i, E_i, F_i, G_i and H_i, in this order. Prove that if $A_1B_1 + E_1F_1 = C_1D_1 + G_1H_1$ and $A_2B_2 + E_2F_2 = C_2D_2 + G_2H_2$ then $A_3B_3 + E_3F_3 = C_3D_3 + G_3H_3$. (*6 points*)

New problems – competition A (see page 481): **A. 809.** Let the lengths of the sides of triangle ABC be denoted by a, b and c using the standard notations. Let S denote the centroid of triangle ABC. Prove that for an arbitrary point P in the plane of the triangle the following inequality is true: $a \cdot PA^3 + b \cdot PB^3 + c \cdot PC^3 \geqslant 3abc \cdot PS$. (Proposed by *János Schultz,* Szeged) **A. 810.** For all positive integers n let r_n be defined as $r_n = \sum_{t=0}^{n} (-1)^t \binom{n}{t} \frac{1}{(t+1)!}$. Prove that $\sum_{n=1}^{\infty} r_n = 0$. **A. 811.** Let A be a given set with n elements. Let $k < n$ be a given positive integer. Find the maximum value of m for which it is possible to choose sets B_i and C_i for $i = 1, 2, \ldots, m$ satisfying the following conditions: (i) $B_i \subset A$, $|B_i| = k$, (ii) $C_i \subset B_i$ (there is no additional condition for the number of elements in C_i), (iii) $B_i \cap C_j \neq B_j \cap C_i$ for all $i \neq j$.

Problems in Physics

(see page 506)

M. 408. Let us make jam jar lids of different sizes collide with each other, such that one of them is at rest and the collision is a head on one. Determine the coefficient of restitution, which characterises the elasticity of the collision.

G. 757. Suppose you have a pair of reversible gloves, both pieces black on the outside and white on the inside. Can you wear them as mismatched gloves? **G. 758.** An object is behind a car not very far from it, and its images can be seen in both side-view mirrors as well as in the rear-view mirror of the car. All the three mirrors are plane mirrors. In which mirror does the driver observe the greatest and the smallest image? In other words compare visual angles of the images. **G. 759.** Four balls of mass m, then another four balls of M and then one more ball of mass m are strung on a horizontal, frictionless, fixed rod, as shown in the *figure.* The balls are close to each other, they are made of some perfectly elastic material and $m < M$. From the left another ball of mass m is coming at a speed of v and collides with the first one of the ball string. After the following collisions which balls remain at rest and what will the direction of the motion of the other balls be? **G. 760.** A 10 cm high aluminium cone is raised slowly out of an aquarium, by means of a thread attached to the apex of the cone. The shape of the aquarium is a rectangular box, and the diameter of the cone is also 10 cm. Initially the base of the cone lies on the bottom of the aquarium, and the cone is totally submerged into the water. The volume of the aquarium is much greater than that of the cone. Plot the graph of the tension in the thread as a function of the displacement of the cone.

P. 5355. Newspaper news (February 24, 2021): *China's Mars probe, Tianwen-1, has been already orbiting Mars and has collected data from the red planet. The furthest point of its parking orbit measured from the surface of Mars is 59 thousand kilometres, while the nearest is 280 kilometres away. The probe makes one revolution around the planet in two Mars days.* By calculation check the accuracy of the value given for the period if the other data are accepted as correct. **P. 5356.** A beam of rectangular cross section lies on the horizontal ground. The horizontal side of the rectangle is L, whilst its vertical side is H. Neglecting air resistance, from which point and how should a grasshopper jump in order to jump over this beam with the least energy? In this case where is the focus of the parabolic path of the leap? **P. 5357.** A uniform-density rod is lying on a horizontal