P. 5174. Egy illegális laboratórium ólomkonténerében olyan sugárzó anyagot találtak, amelyből másodpercenként $2\cdot 10^{14}$ elektron lép ki. A rendőrségi jegyzőkönyvek szerint 53 évvel ezelőtt eltűnt 221 g cézium a közeli kutatóintézetből. Lehet-e a megtalált anyag az akkor eltűnt preparátum, ha azóta csak raktározták? (A cézium felezési ideje 26,6 év.)

(*4 pont*) *Tematikus feladatgyűjtemény,* Szeged

Beküldési határidő: 2019. december 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

❄

MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 69. No. 8. November 2019)

Problems in Mathematics

New exercises for practice – competition K (see page 482): **K. 634.** A sheet of graph paper has a grid of unit squares on it. A rectangle is drawn with sides lying along grid lines. Is it possible to draw a closed broken line in the rectangle along grid lines such that it should never leave the rectangle but it should pass through all the grid points in the interior and on the boundary of the rectangle, if the dimensions of the rectangle are *a*) 2019×2020 units; *b*) 2018×2020 units? If so, determine the length of the possible broken lines, too. **K. 635.** Consider a concave quadrilateral, and draw the diagonal that lies in its interior. The diagonal divides the quadrilateral into two triangles. Prove that the areas of the two triangles are equal if and only if the line of this diagonal bisects the other diagonal. **K. 636.** Find all possible values of the digits x and y for which every nonzero digit occurs the same number of times in the prime factorization of the eight-digit number $\overline{xyxyxyxy}$ in decimal notation. (In making the prime factorization, identical prime factors are not written as a power but written down as separate factors.) **K. 637.** Let us consider the integer 12345678901234567890. . . 1234567890 consisting of 2020 digits. First we remove the digits at every odd position. Then, from the remaining 1010 digits, we remove the digits at every even position. Then, repeating in the same way, from the remaining 505 digits, we remove the digits at every odd position. This alternating process is continued until a single digit remains. Determine this digit. **K. 638.** Fibonacci-type sequences are defined as sequences in which, from the third term onwards, each term is the sum of the preceding two terms. For example, the sequence $1, 1, 2, 3, 5, 8, \dots$ starting with 1, 1 (the Fibonacci sequence itself), and the sequence $1, 3, 4, 7, 11, 18, 29, 47, \dots$ starting with 1, 3 are both Fibonacci-type sequences. Find the Fibonacci-type sequence that contains only positive integers, contains 2010 as a term, and has the largest possible number of terms before 2010.

New exercises for practice – competition C (see page 483): **Exercises up to grade 10: C. 1567.** Find the real solutions of the equation $2x^2-4xy+4y^2-8x+16=0$. (Proposed by *M. Szalai,* Szeged) **C. 1568.** Let D be the midpoint of side AB of a triangle ABC, E the midpoint of side AC, and P, Q the centres of the circumscribed circles of triangles DEB and DEC, respectively (assume that $P\neq Q$). Prove that line PQ is perpendicular to line BC. (Proposed by *D. Hegedűs,* Gyöngyös) **Exercises for everyone: C. 1569.** In a class of 24, there are an odd number of students whose first name is Sophia. When the class is listed in alphabetical order (of family names) and students are numbered in this order, the number of the first Sophia on the list is equal to the number of Sophias in the class, and the number of the third Sophia on the list is three times the number of Sophias in the class. Given that each Sophia on the list is immediately preceded or followed by another Sophia, determine the numbers assigned to all the Sophias on the list. (Based on a problem by *L. Hommer,* Kemence) **C. 1570.** In a hexagon, the measure of each angle is 120°, and the diagonals connecting opposite vertices are equal in length. Prove that the hexagon has rotational symmetry. (Proposed by *K. Fried,* Budapest) **C. 1571.** The positive integers from 1 to n^2 are written in increasing order in an $n \times n$ table: the numbers 1 to n are entered in the first row, $(n+1)$ to $2n$ in the second row; and so on. Prove that the sum of the numbers in one diagonal equals the sum of the numbers in the other diagonal. **Exercises upwards of grade 11: C. 1572.** In a trapezium $ABCD$, let M denote the intersection of diagonals AC and BD, and let N and P denote the centres of the circumscribed circles of triangles ABC and ACD, respectively. Prove that M, N and P are collinear if and only if $ABCD$ is a parallelogram or a cyclic trapezium. **C. 1573.** Show that the sum $12^{2n}+7^{2n-1}+3^{3n}+4^{4n-2}-2^{2n}-11^{2n}$ is divisible by 23 for all positive integers n. (Proposed by *T. Imre,* Marosvásárhely)

New exercises – competition B (see page 484): **B. 5054.** Are there positive integers n and k such that $20^k+19^k=2019^n-10^n$? (*4 points*) (Proposed by *T. Imre,* Marosvásárhely) **B. 5055.** Given a circle k in the plane, determine the locus of the orthocentres of all triangles inscribed in k. (*3 points*) **B. 5056.** Consider the quadratic function $f(x)=x^2+bx+c$ defined on the set of real numbers. Given that the zeros of f are some distinct prime numbers p and q, and $f(p-q)=6pq$, determine the primes p and q, and determine the function f. (*3 points*) (Proposed by *B. Bíró,* Eger) **B. 5057.** Let D and E be points on leg BC of a right-angled triangle with hypotenuse AB such that $\angle DAC=\angle EAD=\angle BAE$. The feet of the perpendiculars dropped from vertex C onto the line segment AD and from point D onto the hypotenuse AB are F and K, respectively. Line CK intersects line segment AE at point H, and the parallel line drawn from point H to the line AD intersects line segment BC at point M. Show that point F is the circumcentre of triangle CHM. (*5 points*) (Proposed by *B. Bíró,* Eger) **B. 5058.** Let P be an arbitrary point in the interior of triangle ABC. Lines AP, BP and CP intersect sides BC, AC, and AB at points A_1, B_1 and C_1, respectively. Prove that $\frac{AP}{A_1P}\cdot\frac{BP}{B_1P}\cdot\frac{CP}{C_1P}\geqslant 8$. (*4 points*) (Proposed by *L. Németh,* Fonyód) **B. 5059.** For a positive integer c, define the sequence $\{a_n\}$ by the following recurrence relation: $a_0=c$ and $a_{n+1}=\left[a_n+\sqrt{a_n}\,\right]$ for $n\geqslant 0$. Prove that if 2019 occurs as a term of the sequence, then no preceding term is a perfect square, but there are infinitely many perfect squares among the following terms. (*5 points*) **B. 5060.** In the plane Σ, given a circle k and a point P in its interior, not coinciding with the center of k. Call a point O of space, not lying on Σ, a *proper projection center* if there exists a plane Σ', not passing through O, such that, by projecting the points of Σ from O to Σ', the projection of k is also a circle, and its center is the projection of P. Show that the proper projection centers lie on a circle. (*6 points*) **B. 5061.** A function $f:\mathbb{R}\to\mathbb{R}$ is called *area preserving* if the area of the triangle formed by the points $\big(a,f(a)\big)$, $\big(b,f(b)\big)$ and $\big(c,f(c)\big)$ is equal to the area of the triangle formed by points $\big(a+x,f(a+x)\big)$, $\big(b+x,f(b+x)\big)$ and $\big(c+x,f(c+x)\big)$ for all $a<b<c$ and x. Which continuous functions f are area preserving? (*6 points*)

New problems – competition A (see page 485): **A. 761.** Let $n\geqslant 3$ be a positive integer. We say that a set S of positive integers is *good* if $|S|=n$, no element of S is a multiple of n, and the sum of all elements of S is not a multiple of n either. Find, in terms of n, the least positive integer d for which there exists a good set S such that there are exactly d nonempty subsets of S the sum of whose elements is a multiple of n.

(Proposed by *Aleksandar Makelov,* Burgas, Bulgaria and *Nikolai Beluhov,* Stara Zagora, Bulgaria) **A. 762.** In a forest there are n different trees (considered as points), no three of which lie on the same line. John takes photographs of the forest such that all trees are visible (and no two trees are behind each other). What is the largest number of orders of in which the trees that can appear on the photos? (Proposed by *Gábor Mészáros,* Sunnyvale, Kalifornia) **A. 763.** Let $k \geqslant 2$ be an integer. We want to determine the weight of n balls. One try consists of choosing two balls, and we are given the sum of the weights of the two chosen balls. We know that at most k of the answers can be wrong. Let $f_k(n)$ denote the smallest number for which it is true that we can always find the weights of the balls with $f_k(n)$ tries (the tries don't have to be decided in advance). Prove that there exist numbers a_k and b_k for which $\left|f_k(n) - a_k n\right| \leqslant b_k$ holds. (Proposed by *Surányi László,* Budapest and *Bálint Virág,* Toronto)

Problems in Physics

(see page 505)

M. 390. By means of a prism made from water, in a simple way split the white light of a LED lamp into components. Write down the method and the result of the observation.

G. 685. The fuel tank of an American car can hold 15 gallons of gasoline. How many miles can the driver of the car go with the car, which was initially filled fully with gasoline, if according to the European catalogue of the car the fuel consumption of the car is 6.5 litres per 100 kilometres? **G. 686.** A 1.5-ton car is staying at rest on a horizontal road. Find the size of the surface the car touches the road, if the pressure in each tyre was adjusted to the value of 2.5 bars at the petrol station. **G. 687.** In a weather forecast not only the temperature but the "feels like" temperature is often given, which might be lower or higher than the temperature measured by the thermometer. When we have just finished a shower in the shower enclosure, then we feel the temperature warmer than the measured temperature in the bathroom, but if we open the door of the enclosure, because we left the towel outside, then our "feels like" temperature is much lower than the real temperature in the bathroom. Explain what is the reason for the fact that despite the temperature in the bathroom is nearly constant, our sense of temperature changes so abruptly. **G. 688.** Some time ago in the movies filmstrips, similar to the ones in children's tale film projectors, were used, just the movie filmstrips were much longer. The length of the filmstrip of a one-minute film was 27 metres. The films were wound on reels, which had a radius of 5.5 cm, and the width of the film roll on it was 12.5 cm. When a film was projected, the film was wound off the reel, called the feed reel, and wound on another reel, called the takeup reel. *a*) What was the number of revolution of the feed reel at the beginning and at the end of the projection of the film? *b*) After the projection the film was wound back from the takeup reel to the feed reel. What was the number of revolution of the takeup reel at the beginning and at the end of the re-wound process if the feed reel was rotated at constant 3 revolutions per second for all the time?

P. 5164. In the same amount of time, the number of complete small-amplitude swings of two simple pendulums are 5 and 10. What are the lengths of the pendulums if one of them is 120 cm longer than the other? **P. 5165.** Circles, which touch each other externally and whose centres are on the same radius of a uniform-density disc of unit radius, are cut out from the disc as shown in the *figure.* The radii of the circles which are cut out are $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$. Where is the centre of mass of the remaining part of the disc if *a*) only the greatest circle, *b*) the two greatest circles, *c*) a lot of circles are cut out from the disc? **P. 5166.** A small object of 30 grams is attached to each end of a 40 cm long rod of an Eötvös pendulum. The extremely light rod is hanging on a very thin metal thread horizontally. A lead ball of mass 100 kg was placed to a distance of three metres