P. 5161. Homogén, $\boldsymbol{B}$ indukcióvektorú, erős mágneses térbe R sugarú, igen hosszú, töltetlen fémhengert helyezünk. A henger tengelyét az indukcióvektorral párhuzamosan rögzítjük, majd akörül ω szögsebességgel forgatni kezdjük. Mekkora felületi töltéssűrűség alakul ki a henger palástján?

(*5 pont*) Közli: *Németh Róbert,* Budapest

P. 5162. Egy szabályos háromszög alapú egyenes hasáb oldallapjai síktükrök. A hasáb a vízszintes alaplapjának súlypontján átmenő, függőleges tengely körül T periódusidővel egyenletes forgómozgást végez. Az egyik oldallapjára vízszintes irányú, a forgástengely felé haladó lézersugár érkezik. A $t=0$ pillanatban a lézersugár merőleges az egyik tükörre. Adjuk meg és ábrázoljuk a visszavert sugárnak a beeső sugárral bezárt szögét az idő függvényében a $0\leqslant t\leqslant T$ időtartamban!

(*4 pont*) Közli: *Zsigri Ferenc,* Budapest

P. 5163. A kobalt 60-as tömegszámú izotópja elektront bocsát ki magából. Milyen atommaggá alakul át?

(*3 pont*) Közli: *Légrádi Imre,* Sopron

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Problems in Mathematics

New exercises for practice – competition K (see page 414): **K. 629.** Seven ducklings are walking towards the lake in a single file: Loppy, Happy, Tappy, Kippy, Boppy, Poppy, and Sippy. They normally walk in the same order every day, but this time they lined up in reverse order. Regarding the present order, the following information is available: The ducklings preceding Loppy could line up in six different ways in a single file. The number of ducklings preceding Boppy is the half of those following him. The number of ducklings between Poppy and Tappy is one fewer than twice the number of those between Sippy and Happy. Happy and Tappy both walk behind Kippy. What is the normal order of the ducklings walking to the lake? **K. 630.** A party is breaking up, and everyone is going home. To say goodbye, every female participant shakes hands with every other female participant, and every male participant shakes hands with every other male participant. During this process, a friend of the host turns up, who shakes hands with everyone he knows, males and females alike. Given that 5 of the participating men also brought their wives along, and 83 handshakes took place altogether, what may be the number of persons known by the friend of the host? **K. 631.** Explain in detail why the following statement is true: if the product of ten positive integers ends in three zeros, then there are six numbers among them such that their product has the same property. **K. 632.** A father had a basket of plums. He gave the plums to his sons in the following way: the first son received 2 pieces plus one nth of the remaining plums, then the second son received 4 pieces plus one nth of the remaining plums, then the third one received 6 pieces plus one nth of the remaining plums, and so on. The rest of the plums the father kept for himself. At the end of this process, it turned out that everyone got the same number of plums. Given that the father had at least 2 sons, what may be the value of n? **K. 633.** Doris thought of an integer, from 3 to 25, inclusive. She told Anna whether the number is a perfect square, whether it is a prime, and whether it is a multiple of 5. From this information, Anna was able to tell which number it was. What may the number be?

New exercises for practice – competition C (see page 415): **Exercises up to grade 10: C. 1560.** Six classes of a school are planning to take trips to the towns of Pécs, Szeged, Debrecen or Miskolc. (Each class is to visit a single town.) Each town must be visited by at least one class. In how many different ways may they select the trip destinations? **C. 1561.** What may be the angles of a triangle, if the triangle formed by the points of tangency of the incircle on the sides is similar to the original triangle? **Exercises for everyone: C. 1562.** Prove that if n^2+1 is divisible by 5 for some integer n, then one of the numbers $(n-1)^2+1$ and $(n+1)^2+1$ is also divisible by 5. **C. 1563.** Let us consider one half of an equilateral triangle (that is, the triangle having angles 30°, 60° and 90°). We create two more triangles by rotating the original one by 30°, and the original one by 60° about its right angle in both cases. Determine the area of the intersection of these 3 triangles. **C. 1564.** A 6×6 square grid is divided into n rectangles of different areas by cutting along grid lines. Give an example of such a dissection for every possible $n>1$. **Exercises upwards of grade 11: C. 1565.** The sides of a trapezium are 2, 3, 5, and 6 units long, in some order. What is the largest possible area of such a trapezium? **C. 1566.** Assume that the probability of a newborn baby being a boy is always p. In families with two children, is it more common to have one boy and one girl than to have two children of the same sex?

New exercises – competition B (see page 416): **B. 5046.** Let $n\geqslant 3$, and consider the graph in which the vertices are the grid points (i,j), where $1\leqslant i,j\leqslant n$, and the distinct points (i,j) and (k,l) are connected by an edge if and only if $i^2+j^2+k^2+l^2$ is divisible by 3. For what values of n is it possible to walk the edges of the graph by traversing each edge exactly once? (*4 points*) (Proposed by *M. Pálfy,* Budapest) **B. 5047.** In a right-angled triangle ABC, point D lies in the interior of leg AC, and point E lies on the extension of hypotenuse AB beyond B. The second intersection of circles ADE and BCE (different from E) is F. Show that $\angle CFD=90^\circ$. (*4 points*) **B. 5048.** The base of a pyramid is a convex polygon, and the areas of the lateral faces are equal. Select an arbitrary point on the base, and consider the sum of the distances of this point from the lateral faces. Prove that the sum is independent of the choice of the point on the base. (*3 points*) (*Croatian problem*) **B. 5049.** Prove that there are infinitely many pairs of positive integers (a,b) for which $2019<\frac{2^a}{3^b}<2020$. (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5050.** Solve the equation $\cos(3x)+\cos^2 x=0$. (*3 points*) **B. 5051.** The sides of quadrilateral $ABCD$ are $AB=8$, $BC=5$, $CD=17$ and $DA=10$. The intersection of diagonals AC and BD is E, and $BE:ED=1:2$. What is the area of the quadrilateral? (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 5052.** Two players, First and Second take turns in writing a number 0 or 1 in the fields of a 19×19 table, initially all blank. When all fields are filled in, they calculate the sum of each row, and the sum of each column. Let the largest row sum be A, and let the largest column sum be B. If $A>B$ then First wins the game. Second wins if $A<B$, and it is a draw if $A=B$.

Does either of the players have a winning strategy? (*6 points*) **B. 5053.** Let G denote the inscribed sphere of a tetrahedron $ABCD$, and let G_A be the escribed sphere touching the face BCD. Let T be the tetrahedron formed by the points of tangency of G on the planes of the faces, and let T_A be the tetrahedron formed by the points of tangency of G_A on the planes of the faces. Show that $\frac{V^3(T)}{V^3(T_A)} = \frac{V^2(G)}{V^2(G_A)}$ holds for the volumes of the tetrahedra and the spheres. (*6 points*)

New problems – competition A (see page 418): **A. 758.** In quadrilateral $ABCD$, $AB = BC = \frac{1}{\sqrt{2}}DA$, and $\angle ABC$ is a right angle. The midpoint of side BC is E, the orthogonal projection of C on AD is F, and the orthogonal projection of B on CD is G. The second intersection point of circle BCF (with center H) and line BG is K, and the second intersection point of circle BHC and line HK is L. The intersection of lines BL and CF is M. The center of the Feurbach circle of triangle BFM is N. Prove that $\angle BNE$ is a right angle. (Proposed by *Zsombor Fehér,* Cambridge) **A. 759.** We choose a random permutation of numbers $1, 2, \ldots, n$ with uniform distribution. Prove that the expected value of the length of the longest increasing subsequence in the permutation is at least $\sqrt{n}$. (Proposed by *László Surányi,* Budapest) **A. 760.** An illusionist and his assistant are about to perform the following magic trick. Let k be a positive integer. A spectator is given $n = k! + k - 1$ balls numbered $1, 2, \ldots, n$. Unseen by the illusionist, the spectator arranges the balls into a sequence as he sees fit. The assistant studies the sequence, chooses some block of k consecutive balls, and covers them under her scarf. Then the illusionist looks at the newly obscured sequence and guesses the precise order of the k balls he does not see. Devise a strategy for the illusionist and the assistant to follow so that the trick always works. (The strategy needs to be constructed explicitly. For instance, it should be possible to implement the strategy, as described by the solver, in the form of a computer program that takes k and the obscured sequence as input and then runs in time polynomial in n. A mere proof that an appropriate strategy exists does not qualify as a complete solution.) (Proposed by *Nikolai Beluhov,* Bulgaria, and *Palmer Mebane,* USA)

Problems in Physics

(see page 442)

M. 389. Attach a piece of thread to an egg and place the egg into a cylinder-shaped container. Pour water into the container such that it covers the egg, and then carefully pull out the egg from the water with the help of the thread. Measure how the tension in the thread depends on the displacement of the egg. Determine the work done, while the egg was pulled out. Does this work depend on the cross section of the container?

G. 681. An artefact made of pure gold was found and dug out in good condition during an excavation at an archaeological site. The 2 litre cylinder-shaped artefact has thin uniform walls, and has no top base. The inner diameter and the inner height of the cylinder are equal. When the empty cylinder is carefully placed into water such that its symmetry axis is vertical all the time, then it reaches an equilibrium position when the level of the water outside is at $\frac{5}{8}$-th of the inner height of the cylinder. Determine the width of the wall of the artefact. **G. 682.** The data in the *figure* can be read on the label of an old boiler. *a*) What is the efficiency of the boiler if during the warm-up time (5 hours) the boiler heats up water from a temperature of 15 °C to 75 °C? *b*) Today this boiler is used at a voltage of 230 V. To what time did the warm-up time of the boiler decreased? **G. 683.** We have two alike (red) resistors and also two alike (blue) resistors.