minden pillanatban megegyezik. Legfeljebb mekkora lehet a dugattyú elmozdulása, ha a higany, a tartály és a dugattyú hőtágulásától eltekintünk?

(*6 pont*) Közli: *Berke Martin,* Zalaegerszegi Zrínyi Miklós Gimnázium

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Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

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Problems in Mathematics

New exercises for practice – competition K (see page 479): **K. 599.** Write the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles, so that the sum of the four numbers along any straight line should be the same, and the sum of the numbers at the six points of the star should also be the same number. A few numbers are already entered. Find all possible arrangements. **K. 600.** If one digit of a three-digit number is omitted, a two-digit number will be obtained. By omitting one digit of that two-digit number, a one-digit number will result. What may be the initial three-digit number so that the sum of the three-digit number, the two-digit number and the final one-digit number is 1001? **K. 601.** The sides of a square $PQRS$ inscribed in an acute-angled triangle ABC are 4 cm long, vertices P and Q lie on side AB, vertex R lies on side BC, and vertex S lies on side AC. Given that the length of side AB is 8 cm, what is the area of the triangle? **K. 602.** Andrew and Paul are playing a game. The winner is always awarded x points and the loser always gets y points (where $x > y$ are integers). There is no draw. After a few rounds, we observe that Andrew has 30 points and Paul has 25 points since Paul has only won twice. How many points are awarded to the winner? **K. 603.** I have a two-digit number in mind. Let S denote the sum of the digits, and let P denote their product. What may be my number if it is equal to $P+S$?

New exercises for practice – competition C (see page 480): **Exercises up to grade 10: C. 1504.** A 3×3 table is filled in as shown. If the greatest common divisor of any set of n entries of the table is n, it is allowed to rearrange those entries so that none of them stay in place. With an appropriate succession of such steps, is it possible to achieve that the final arrangement of the numbers is a reflection of the original arrangement in one diagonal? In the other diagonal? **C. 1505.** Consider the circumscribed circles of all the black fields of a chessboard. What fraction of the total area of the 64 fields is covered by these disks altogether? **Exercises for everyone: C. 1506.** Solve the equation $p^q+1=q^p$, where p, q denote positive prime numbers. **C. 1507.** The perpendicular bisectors of the legs of an obtuse-angled isosceles triangle divide the base into three equal parts. Find the measures of the angles. **C. 1508.** Determine the value of xy, given that $x+y=1$ and $x^3+y^3=\frac{1}{2}$. **Exercises upwards of grade 11: C. 1509.** A company selling teabags has placed gift vouchers in 10% of the boxes. If 10 boxes are bought, what is the probability of finding more than 1 voucher? **C. 1510.** The base radii of a right circular truncated cone are 8 cm and 5 cm. The slant height is 12 cm. If the truncated cone is laid on its side and rolled, it will trace out a circular ring in the plane. Determine the radii of the inner and outer circles of the ring, and find the number of times the truncated cone rotates about its axis while it rolls around and returns to its starting position.

New exercises – competition B (see page 481): **B. 4982.** The diagonals AC and BD of a convex kite $ABCD$ intersect at point E such that $AE<CE$. The midpoint of diagonal AC is F. The circles ABE and CDE intersect again at M. Show that $\angle EMF=90^\circ$. (*3 points*) **B. 4983.** Find the real solutions of the equation $x^2+2x-3-\sqrt{\frac{x^2+2x-3}{x^2-2x-3}}=\frac{2}{x^2-2x-3}$. (*4 points*) (Proposed by *L. Laczkó* and *J. Szoldatics,* Budapest) **B. 4984.** Prove that for any positive integer x, there exists a positive integer y such that x^3+y^3+1 is divisible by the number $x+y+1$. Is there a positive integer x for which there are infinitely many y with this property? (*4 points*) (Proposed by *L. Surányi,* Budapest) **B. 4985.** Given that any three out of four lines determine a triangle, prove that the orthocentres of the four triangles are concurrent. (*5 points*) **B. 4986.** Consider the 64 points of the space for which each of the three coordinates is 1, 2, 3 or 4. Kate and Peter are playing a three-dimensional tic-tac-toe game on this set of points. Kate starts the game by selecting any point and colouring it blue. In the second step, Peter selects a different point and colours it red. Then they take turns by selecting further points and colouring them in blue or red. Whoever first completes a collinear set of four points of their own colour will win the game. Show that it makes no difference for Kate whether she starts by colouring the point $(1,1,2)$ or the point $(2,2,1)$ blue in the first step. (*5 points*) (Proposed by *D. Benkő,* South Alabama) **B. 4987.** The circumcentre of an acute-angled scalene triangle ABC is O, its orthocentre is M, the foot of the altitude drawn from vertex A is D, and the midpoint of side AB is F. The ray drawn from F through M intersects the circumcircle of triangle ABC at G. *a*) Prove that the points A, F, D and G are concyclic. *b*) Let K denote the circle in *a*), and let E be the midpoint of line segment CM. Prove that $EK=OK$. (*5 points*) (Proposed by *B. Bíró,* Eger) **B. 4988.** In an $(m+2)\times(n+2)$ table, we cut out the four 1×1 "corners". Arbitrary real numbers are written in each field of the first and last rows, and in the first and last columns of the truncated table obtained in this way. Prove that it is possible to fill in the remaining $m\times n$ "interior" of the table in a unique way with real numbers such that every number is the arithmetic mean of the four adjacent numbers. (*6 points*) (Competition problem from Iran) **B. 4989.** The midpoints of sides BC, CA and AB of a triangle ABC are D, E and F, respectively. Let S denote the centroid of the triangle. Assume that the perimeters of triangles AFS, BDS are CES equal. Show that triangle ABC is equilateral. (*6 points*)

New problems – competition A (see page 482): **A. 734.** For an arbitrary positive integer m, not divisible by 3, consider the permutation $x\mapsto 3x \pmod{m}$ on the set $\{1,2,\dots,m-1\}$. This permutation can be decomposed into disjoint cycles; for instance, for $m=10$ the cycles are $(1\mapsto3\mapsto9\mapsto7\mapsto1)$, $(2\mapsto6\mapsto8\mapsto4\mapsto2)$ and $(5\mapsto5)$. For which integers m is the number of cycles odd? **A. 735.** For any function $f:[0,1]\to[0,1]$, denote by $P_n(f)$ the number of fixed points of the function $\underbrace{f\big(\dots f}_{n}(x)\dots\big)$, i.e., the number of points $x\in[0,1]$ satisfying $\underbrace{f\big(\dots f}_{n}(x)\dots\big)=x$. Construct a piecewise linear, continuous, surjective function $f:[0,1]\to[0,1]$ such that for a suitable number $2<A<3$, the sequence $\frac{P_n(f)}{A^n}$ converges. (Based on the *8th problem of the Miklós Schweitzer competition,* 2018) **A. 736.** Let P be a point in the plane of triangle ABC. Denote the reflections of A, B, C about P by A', B' and C', respectively. Let A'', B'', C'' be the reflections of A', B', C' over the lines BC, CA and AB, respectively. Let the line $A''B''$ intersect AC at

A_b and let $A''C''$ intersect AB at a point A_c. Denote by ω_A the circle through the points A, A_b, A_c. The circles ω_B, ω_C are defined similarly. Prove that ω_A, ω_B, ω_C are coaxial, i.e., they share a common radical axis. (Proposed by *Navneel Singhal,* Delhi and *K. V. Sudharshan,* Chennai, India)

Problems in Physics

(see page 505)

M. 381. Make a cylinder from a sheet of A4-size paper (or some part of it) by gluing it. Roll it down from the top of an inclined plane, which is on a table such that its lower end is just at the rim of the table. Measure how far from the vertical projection of the rim of the table on the floor the cylinder reaches the floor. (The cylinder should roll without sliding.) How does this distance depend on the diameter of the cylinder? Compare your results with the horizontal displacement of a small object (e.g. a coin) which slides without rolling along the slope and falls from the end of the slope.

G. 649. If we move in a steam bath (for example we begin to fan ourselves with the arms), then we feel the 40–60 °C vapour much hotter, and it may feel like our skin is burning. Why? **G. 650.** Aaron played with his ruler and rubber. He put the small rubber to one end of the 30-cm long ruler and slowly slid the ruler along the tabletop, perpendicular to the rim of the table, such that the end of the ruler was sticking out from the table. He measured that when the ruler is sticking out more than 11 cm it falls down. What is the ratio of the masses of the ruler and the rubber? **G. 651.** On the hull of ocean liners there is a mark called the *Plimsoll line.* This shows the legal limit to which the ship may be loaded, indicating the maximum draft of the ship in specific water types and temperatures.* The topmost TF line marks the draft of the hull when the liner is in tropical fresh water (its density is 996 kg/m^3), whilst the F line below marks the maximum draft in case of fresh water in temperate zone (its density is 999 kg/m^3). On a specific ocean liner the distance between these two lines is 7 cm. What is the density of winter seawater if its mark on the hull of this liner is 21 cm below the topmost line? **G. 652.** An object starts from rest and moves along a straight line such that its acceleration increases uniformly in time, from the value of zero it increases by 2 m/s^2 in each second. What is the speed of the object 4 s after it started to move?

P. 5067. Two light springs, whose spring constants are D_1 and D_2 are put onto a frictionless rod. One end of each spring is fixed and the distance between their other ends is d. There is also a small object of mass m on the rod, and with this small object one of the springs is compressed by x, and then the object is released. How much time elapses until the object of mass m reaches its initial position? Does this time depend on the spring it was started from? **P. 5068.** A small comet of mass m, which can be considered point-like, approaches a spherical planet of mass M, and of radius R ($m \ll M$). The speed of the comet very far from the planet is v_0, and if the gravitational field of the planet did not exert any force on the comet, it would pass the planet at a distance of d from the centre of the planet ($d > R$). What is the minimum value of v_0, at which the comet does not hit the planet? (Apart from the planet and the comet, the gravitational fields of any other celestial objects can be neglected.) **P. 5069.** A solid cylinder of mass M and of radius R was placed onto a slope of angle of inclination of α. The cylinder is attached to the top end of the inclined plane by means of a horizontal thread as shown in the *figure.* Next to the object there is another cylinder of mass m and of radius r. Friction between the two cylinders is negligible, and the cylinder of mass M is not rising. What is the least value of the coefficient of static friction between the slope and the cylinder of radius R if the

*TF = Tropical Fresh Water; F = Fresh Water; T = Tropical Seawater; S = Summer Seawater; W = Winter Seawater; WNA = Winter North Atlantic.