d) Miből lehetne még hangszerhúrt készíteni? Keressünk megfelelő anyagot a Függvénytáblázatban és az Interneten!

A gitárhúr rezgő részének hossza (menzúrahossz) 64 cm. Az acélhúr anyagának sűrűsége $7{,}8\cdot 10^3$ kg/m^3.

(*6 pont*) Közli: *Vladár Károly,* Kiskunhalas

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Problems in Mathematics

New exercises for practice – competition K (see page 92): **K. 577.** Xavier picked three cards out of a deck of French cards, and placed them on the table in a row. He gave the following information on the cards picked: – One of the three cards is a king, and immediately to the right of this king there are one or two queens. – One of the three cards is a queen, and immediately to the left of this queen there are one or two queens. – One of the three cards is a heart, and immediately to the left of this heart there are one or two spades. – One of the three cards is a spade, and immediately to the right of this spade there are one or two spades. What cards may there be on the table, from left to right? **K. 578.** The positive integers 1 to n are written in the fields of the upper row of a $2\times n$ table, in increasing order. The same numbers are written in the lower row, in decreasing order. How many positive integers n smaller than 50 are there for which every number in the upper row is relatively prime to the number directly below? **K. 579.** We form 100 pairs out of 105 girls and 95 boys in a random way. The boy-and-boy pairs shake hands, the girl-and-girl pairs give each other a hug, and the mixed pairs start to dance. Show that the number of handshakes taking place is 5 less than the number of hugs. **K. 580.** For what right-angled triangles is it true that $x>2(z-y)$, provided $z>y\geqslant x$? **K. 581.** Find all four-digit square numbers of the form ABBA. **K. 582.** How long may a word be if its letters can be ordered in exactly 180 ways? Give an example of a meaningful English word of this type.

New exercises for practice – competition C (see page 93): **Exercises up to grade 10: C. 1462.** The first term of an arithmetic sequence is $a_1=3$, and its common difference is 9. Prove that for every natural number k, the number $3\cdot 4^k$ occurs among the terms. **C. 1463.** M is an interior point of a regular triangle ABC. The feet of the perpendiculars dropped from M onto the sides AB, BC and CA are H, K and P, respectively. Prove that *i*) $|AH|^2+|BK|^2+|CP|^2=|HB|^2+|KC|^2+|PA|^2$; *ii*) $|AH|+|BK|+|CP|=|HB|+|KC|+|PA|$. (*Mathematical Competitions in Croatia*) **Exercises for everyone: C. 1464.** We say that a natural number B *can be read out of* a larger natural number A, if it is possible to erase some of the digits of A so that B is obtained by reading the remaining digits, without changing their order.

What is the smallest natural number, such that every three-digit number can be read out of it? **C. 1465.** Let M denote the intersection of the lines PS and RT passing through the vertices of a regular triangle PQR and a square $QRST$. Show that triangle PTM is isosceles. **C. 1466.** A committee had twelve meetings during the course of a year. At every meeting, there were 10 members of the committee present. Any pair of members were present at most once together. What is the minimum possible number of members on the committee? **Exercises upwards of grade 11: C. 1467.** Let A and B denote the intersection of the circle of radius $2r$ centred at O, and the circle of radius $r+1$ passing through O. How long may r be if the line segment AB is the diameter of the smaller circle? **C. 1468.** Prove that for all non-negative numbers a and b $\frac{1}{2}(a+b)^2+\frac{1}{4}(a+b)\geqslant a\sqrt{b}+b\sqrt{a}$. When will the equality hold?

New exercises – competition B (see page 94): **B. 4930.** Every inhabitant of a village belongs to one of three religious faiths: they either worship the Sun God, the Moon God or the Earth God. Regulations of these faiths require that a shrine should have the minimum possible total distance from all the houses of the village (whatever the faith of those living in the houses). Given that the worshippers of the Sun God already have a shrine in the village, and those of the Moon God have one, too, show that the worshippers of the Earth God can also build a shrine for themselves. (The village lies on flat terrain, and the shrines and the houses of the village can be considered pointlike.) (*3 points*) **B. 4931.** Prove that if a, b, c are the sides of a triangle then $\frac{a^2(b+c)+b^2(a+c)}{abc}>3$. (*3 points*) **B. 4932.** The Great Bestiary of Wonderland features a dragon for every week of the year. All dragons have different ages. The youngest dragon, named Aloysius has 13 heads. The second youngest one, Bartholomeus has 14 heads, ... (and so on, each dragon in the order of their ages has one more head than the previous one). The oldest dragon, Zebulon has 64 heads. Wonderland monks are writing the Giant Codex of Dragon Tales. A tale may only be included in the Codex if the total number of heads of all the dragons in the tale is exactly 1001. For every pair of tales, the sets of dragons mentioned in the tales are different. Which of the 13-headed Aloysius and the 14-headed Bartholomeus will appear in more tales when the monks are finished with writing down all possible tales? (*5 points*) **B. 4933.** Find the area of a regular triangle of maximum perimeter inscribed in a unit square. (*4 points*) **B. 4934.** For any positive integers n and k, let $f(n,k)$ denote the number of unit squares cut in two by a diagonal of an $n\times k$ lattice rectangle. How many number pairs n, k are there such that $n\geqslant k$, and $f(n,k)=2018$? (*4 points*) **B. 4935.** The given circles ω_1 and ω_2 lie inside an angle of vertex O, touching the arms. A ray drawn from point O intersects circle ω_1 at points A_1 and B_1, and circle ω_2 at points A_2 and B_2, such that $OA_1<OB_1<OA_2<OB_2$ (see the *diagram*). Circle γ_1 touches the circle ω_1 on the inside, and also touches the tangents drawn to circle ω_2 from point A_1. Similarly, circle γ_2 touches the circle ω_2 on the inside, and also touches the tangents drawn to circle ω_1 from point B_2. Prove that the radii of the circles γ_1 and γ_2 are equal. (*5 points*) (*Kvant*) **B. 4936.** Let AB be a fixed chord that is not a diameter in a circle k. The midpoint of AB is F. Let P be a point on the circle k, different from A and B. Let the line PF intersect circle k again at X, and let Y be the reflection of X in the perpendicular bisector of AB. Prove that there exists a point in the plane that lies on the line PY for all P. (*5 points*) (Proposed by *L. Surányi,* Budapest) **B. 4937.** In the plane, a set of lattice quadrilaterals with the following property is selected: however the lattice points are coloured with a finite number of colours, there will always be a selected quadrilateral whose vertices all have the same colour. Prove that there are infinitely many selected lattice quadrilaterals, no two of which have a vertex in common. (*6 points*) (Proposed by *L. Surányi,* Budapest) **B. 4938.** It is known that it is possible to draw the complete graph with 7 vertices on

the surface of a torus (see the Császár polyhedron, for example). 7 points are marked on the side of a mug. We want to connect each pair of points with a curve, so that the curves have no interior points in common. What minimum number of these curves need to lead across the handle of the mug? (*6 points*)

New problems – competition A (see page 96): **A. 716.** Let ABC be a triangle and let D be a point in the interior of the triangle which lies on the angle bisector of $\angle BAC$. Suppose that lines BD and AC meet at E, and that lines CD and AB meet at F. The circumcircle of ABC intersects line EF at points P and Q. Show that if O is the circumcenter of DPQ, then OD is perpendicular to BC. (Proposed by: *Michael Ren,* Andover, Massachusetts, USA) **A. 717.** We say that a positive integer is *lazy* if it has no prime divisor greater than 3. Prove that there are at most two lazy numbers strictly between two consecutive square numbers. (Proposed by: *Zoltán Gyenes* and *Géza Kós,* Budapest) **A. 718.** Let $\mathbb{R}[x,y]$ denote the set of two-variable polynomials with real coefficients. We say that the pair (a,b) is a *zero* of the polynomial $f\in\mathbb{R}[x,y]$ if $f(a,b)=0$. If polynomials $p,q\in\mathbb{R}[x,y]$ have infinitely many common zeros, does it follow that there exists a non-constant polynomial $r\in\mathbb{R}[x,y]$ which is a factor of both p and q?

Problems in Physics

(see page 122)

M. 375. Measure how the force exerted by the prongs of a clothes peg depends on the angle between the prongs.

G. 625. A spider is crawling at a uniform speed of 1 mm/s along the 1.5-metre minute hand of the tower clock, from the centre of the clock towards the end of the minute hand. The spider starts exactly at 12 o'clock. *a*) What is the time shown by the clock, when the spider reaches the end of the minute hand? Reaching the end of the minute hand the spider descends on a self-made thread attached to the end of the minute hand. *b*) At what rate should the silk of the thread be made in order that the spider reach its starting position exactly at 13? *c*) How far was the spider from the centre of the clock at 12:45? **G. 626.** What is the period of that simple pendulum of length ℓ the thread of which bumps into a peg at the midpoint of the thread when the bob passes the equilibrium position? (The maximum angle that the thread differs from the vertical is small.) **G. 627.** At what height above the surface of the Earth will the gravitational force exerted on an object be exactly the same as that of on the surface of the Moon? **G. 628.** At the same time every morning it can be observed that Venus gets closer to the Sun. Where eventually will Venus pass the Sun, „in front of" or „behind" it?

P. 5001. Same as exercise **G. 628. P. 5002.** The centre of the Earth moves along a bit wavy elliptical path about the Sun. *a*) What is the reason for this waviness? *b*) Approximately what is the amplitude of the wave? **P. 5003.** Two simple pendulums, both having a length of ℓ can swing in parallel, vertical planes, one right behind the other. Their shadows are projected perpendicularly to a wall, and every once in a while the shadows cross each other. Both pendulums are displaced by the same (small) angle and they are released at a time difference of t_0. (t_0 is smaller than the period of the pendulums.) *a*) When do their shadows meet first? *b*) When does the n-th encounter occur? **P. 5004.** The two ends of an 8-meter-long flexible thread are fixed at the same height at a distance of 4 m from each other. A 0.5 kg object is strung on the thread and can move frictionlessly along it. The thread is held tight and the object is released without initial speed such that the starting point and the ends of the thread are collinear. What is