quadrilateral $ABCD$ are E and F, respectively. The line segments AE, EF and AF divide the quadrilateral into four triangles whose areas are four consecutive integers. What is the maximum possible area of triangle ABD? (*5 points*) (Proposed by *S. Róka*, Nyíregyháza)
B. 4907. 1×1 squares are placed on a rectangle of dimensions $a \times b$, with sides parallel to the sides of the rectangle. Prove that the maximum number of such squares without overlaps is $[a] \cdot [b]$ (where $[x]$ stands for the greatest integer not greater than the number x). (*5 points*) **B. 4908.** Let C denote an arbitrary point on the circumference of a circle of diameter AB. Let T be the perpendicular projection of point C onto the line segment AB. Draw the circle of centre C that passes through T. Let the intersections of the two circles be P and Q. Prove that line PQ bisects the line segment CT. (*4 points*) (*Kvant*) **B. 4909.** Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that the equation $x \cdot f(y) - y \cdot f(x) = f\left(\frac{y}{x}\right)$ holds for all $x \neq 0$ and y. (*6 points*) (*Kvant*) **B. 4910.** Let P, Q, R and S denote points on the lines of the sides of a square $ABCD$ such that $AP = BQ = CR = DS$, as shown in the *figure*. Starting from an arbitrary interior point X of side AB, let the line PX intersect line BC at Y, let QY intersect line CD at Z, let RZ intersect line DA at V, and finally let SV intersect line AB at X'. Prove that if X' and X coincide then $XYZV$ is a square. (*5 points*) **B. 4911.** We have placed chessmen on a 8×8 chessboard. There is an odd number of chessmen standing in every row, and in every column. Prove that the total number of chessmen on the black fields of the chessboard is even. (*5 points*)

New problems – competition A (see page 480): **A. 707.** 100 betyárs stand on the Hortobágy plains. Every betyár's field of vision is a 100 degree angle. After each of them announces the number of other betyárs they see, we compute the sum of these 100 numbers. What is the largest value this sum can attain? **A. 708.** Let S be a finite set of rational numbers. For each positive integer k, let $b_k = 0$ if we can select k (not necessarily distinct) numbers in S whose sum is 0, and $b_k = 1$ otherwise. Prove that the binary number $0.b_1b_2b_3 \ldots$ is a rational number. Would this statement remain true if we allowed S to be infinite? **A. 709.** Let $a > 0$ be a real number. Find the minimal constant C_a for which the inequality $C_a \sum_{k=1}^n \frac{1}{x_k - x_{k-1}} > \sum_{k=1}^n \frac{k+a}{x_k}$ holds for any positive integer n and any sequence $0 = x_0 < x_1 < \cdots < x_n$ of real numbers.

Problems in Physics
(see page 506)

M. 372. Make a sandglass from a cylinder-shaped plastic (PET) water bottle. Make a small hole (of diameter approximately 8-10 mm) on the bottle cap through which the dry sand can flow out. Measure how the amount of the out-flowing sand in a unit of time depends on the height of the sand in the bottle.

G. 613. A vehicle undergoes circular motion at a constant speed of 72 km/h. How much time elapses until it gets back to the same point if its acceleration is 1.6 m/s²?
G. 614. Disks of equal mass of m were attached to the ends of a negligible-mass spring of spring constant D. The disks and the spring in unstretched position are placed to an air-cushioned table, and they are given a velocity of v_0 in the direction of the axis of the spring. At a certain instant the disk at the back is suddenly stopped and held at rest. *a*) How much time elapses until the other disk turns back? *b*) What is the greatest extension of the spring and at most how much potential energy is stored in the spring? *Data:* $D = 16$ N/m, $m = 0.25$ kg, $v_0 = 2$ m/s. **G. 615.** A container, filled with water halfway, is sliding down (with some acceleration) along a long enough slope of angle of elevation of $30°$. What is the angle between the surface of the water and the plane of the slope, if friction is negligible? **G. 616.** The radius of a thin-walled, negligible-mass gymnastic ball is 30 cm, the pressure of the enclosed air is $1.1 \cdot 10^5$ Pa, and the ambient air pressure is $1.0 \cdot 10^5$ Pa. How much does the volume of the ball decrease when a 50 kg

person sits upon it with her total weight, if in this case the ball touches the ground along a circle of radius 10 cm? The temperature of the air is constant.

P. 4970. A vehicle travels from city A to city B. During the first part of the journey its average speed is v_1 whilst in the rest of the journey it is v_2. By what factor is the length of the second part of the journey longer than that of the first part if the average speed calculated for the whole journey is a) the arithmetic mean; b) the geometric mean; c) the harmonic mean; d) the weighted arithmetic mean of ratio $1 : k$ of the average speeds calculated for the two parts of the journey? **P. 4971.** A container, filled with water halfway, is sliding down with some acceleration along a long enough slope of angle of elevation of $30°$. What is the angle between the plane of the slope and the surface of the water if the coefficient of kinetic friction between the container and the slope is 0.2? **P. 4972.** One end of a piece of unstretchable, light thread of length ℓ is suspended, whilst the other end is attached to a small ring which can slide without friction along a horizontal rod – at a height of $d < \ell$ above the point of suspension. The thread is held tight, and next to the rod a small object of mass m is placed by means of a pulley onto the thread, and then the system is released. a) What will the speed of the object be at the lowermost point of its path? b) Along what kind of curve does the object move? c) What is the extension in the thread at the lowermost point of the path? **P. 4973.** During a lecture given by Baintner, Géza (1892–1980) there was an experiment shown, that when three rubber strings are tied in a Y shape, and the two strings at the two symmetrical ends of the Y are vibrated totally out of phase, then the two waves cancel each other, and the third piece of the Y stayed at rest. Question: *What happens to the energy of the two waves?* **P. 4974.** A thermally insulated cylinder is attached to a trolley. In the cylinder an easily moveable, and thermally insulated piston of mass $M = 20$ kg separates two chambers of volume $V_0 = 50$ L, both containing air at a temperature of $T_0 = 300$ K, and at a pressure of $p_0 = 10^5$ Pa. The trolley moves towards a wall with speed $v = 10$ m/s, with which it collides totally inelastically. At most what may the temperature of the air be in that chamber which is closer to the wall? **P. 4975.** During an experiment carried out in a laboratory on the Earth, a small object of mass m and of charge Q is released in uniform horizontal magnetic field of induction B. (It can be assumed that $mg < QBc$ where c is the speed of light.) The motion of the object is observed until it reaches its lowermost position. a) What is the greatest speed of the object? b) To what depth does it sink? c) What is its average speed along the horizontal direction? d) What is the acceleration of the object at the lowermost point of its path? **P. 4976.** Three small balls are placed along a straight line, such that initially they do not move, and the distance between two neighbouring balls is d. The masses and the charges of the balls are m, $2m$, $5m$ and q, q, $2q$, respectively. a) What is the distance between the balls, and what is their velocity when a very short time of t_0 elapses after the balls start to move? b) What is the speed of the balls after a long enough time? (Apart from the electrostatic forces any other forces can be neglected.) **P. 4977.** The capacitors in the circuit shown in the *figure* are uncharged, before turning the switch on. At a certain moment the switch is closed. (Neglect the internal resistance of the voltage supply, the capacitance of the wires and the resistors, and the inductance of any of the elements in the circuit.) *Sketch* a graph of the voltage across the capacitors as a function of time. *Data:* $C_1 = 150$ μF, $C_2 = 50$ μF, $R_1 = 40$ kΩ, $R_2 = 10$ kΩ, $U_0 = 100$ V. **P. 4978.** In the engine of an ion propulsion rocket positive heavy ions are accelerated, such that these ions flow through the nozzle, and this accelerates the rocket. There is an electron accelerator on the rocket as well, why is it necessary? **P. 4979.** A drop of mercury of radius R and of surface tension α is floating in weightlessness. If the drop is placed into weak, uniform electric field of magnitude E_0, then the drop will extend in the direction of the electric field, its shape is nearly a rotational ellipsoid. Estimate the length of the extended mercury drop.