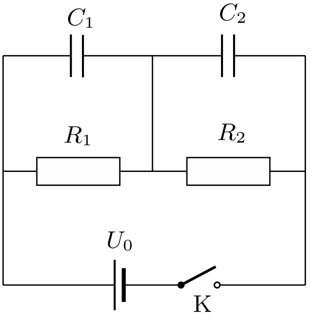
b) Mekkora lesz a golyók sebessége elegendően hosszú idő múlva?

(Az elektrosztatikus erőkön kívül minden más erőhatás elhanyagolható.)

(*5 pont*) *A Kvant nyomán*

P. 4977. Az *ábrán* látható kapcsolásban a kapcsoló zárása előtt a kondenzátorok töltetlenek. Egy adott pillanatban zárjuk a kapcsolót. (Az áramforrás belső ellenállásától, a vezetékek és az ellenállások kapacitásától, továbbá a körben lévő elemek induktivitásától tekintsünk el.)



Ábrázoljuk *vázlatosan* a kondenzátorok feszültségét az idő függvényében!

Adatok: $C_1 = 150\ \mu\text{F}$, $C_2 = 50\ \mu\text{F}$, $R_1 = 40\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, $U_0 = 100\ \text{V}$.

(*5 pont*) *Nagy László* (1931–1987) feladata

P. 4978. Az ionrakéta hajtóművében pozitív töltésű nehézionokat gyorsítanak fel, ezek áramlanak ki a fúvókán keresztül, ettől gyorsul fel a rakéta. Ugyanekkor elektrongyorsítót is beszerelnek az ionrakétába, erre miért van szükség?

(*3 pont*) *Némedi István* (1932–1998) feladata

P. 4979. A súlytalanság állapotában egy R sugarú, α felületi feszültségű higanycsepp lebeg. Ha a cseppet gyenge, E_0 térerősségű, homogén elektromos térbe helyezzük, a csepp a térerősség irányában kissé megnyúlik, alakja forgási ellipszoiddal közelíthető. Adjunk becslést, mekkora lesz a megnyúlt higanycsepp hossza.

(*6 pont*) Közli: *Vigh Máté,* Budapest

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Problems in Mathematics

New exercises for practice – competition K (see page 476): **K. 559.** How many at most six-digit numbers are there in which each of the digits 1, 2, 3, 4, 5 occurs exactly once? **K. 560.** There were 30 candidates taking an exam. The average score of those who failed was 60, and the average score of those who passed was 84. The average score of all candidates was 80. How many of them passed the exam? **K. 561.** A novel was published in three volumes. The page numbering started with 1 in the first volume, and in the second and third volumes it continued where the previous volume ended. The second volume was 50 pages thicker than the first one, and the third volume was 1.5 times as thick as the second volume. The sum of the first page numbers in the three volumes was 893. How many pages long is the entire novel? How many digits were used altogether in numbering the pages? **K. 562.** Alice went shopping. She only had 10-forint coins (HUF, Hungarian currency) and 1000-forint notes on her, at least one of each. When she had spent half her money, she noticed that she only had 10-forint coins and 1000-forint notes again. She had as many 10-forint coins as the number of 1000-forint notes she had set out with, and she had half as many 1000-forint notes as the initial number of her 10-forint coins. Given that she had spent the least amount of money that meets the given conditions, how many forints had she spent? **K. 563.** A disc of radius 3 cm has been cut out of each corner of a square plate of side 18 cm as shown in the *figure.* The small pieces falling off at the corners were thrown away. What is the area of the remaining part of the plate? **K. 564.** A spider wears 8 identical socks and 8 identical shoes on its feet. (There needs to be a sock and a shoe on every foot.) In how many different orders may the spider put on his socks and shoes in the morning, provided that for any given foot, the sock needs to be put on before the shoe, but not necessarily directly before that. (Two orders are only distinguished by the order of the feet.)

New exercises for practice – competition C (see page 477): **Exercises up to grade 10: C. 1441.** A coffee shop serves coffee specials made from various ingredients. For any item selected from the menu there exist exactly three others that each have some ingredient in common with the selected item. If two menu items have no ingredient in common then there exists a third one that has an ingredient in common with each of the two. What is the maximum possible number of items on the menu of cofee specials? **C. 1442.** The sides a, b and c of a triangle satisfy $1=\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}$. Prove that $r\cdot R=\frac{1}{2}$, where r is the inradius and R is the circumradius. (Proposed by *Zs. M. Tatár,* Felsőgöd) **Exercises for everyone: C. 1443.** In how many different ways is it possible to represent 2017^3 as a sum of consecutive positive odd numbers? (Based on the idea of *L. Hommer,* Kemence) **C. 1444.** Solve the following inequality: $x^4-4x^3+8x^2-8x\leqslant 96$. **C. 1445.** The movie "The Englishman Who Went up a Hill but Came down a Mountain" is set in a Welsh village where the neighbouring mountain was designated as a hill by cartographers measuring its height. The villagers were too proud of their mountain to accept this. So they decided to raise its height from 984 feet to 1004 feet. They would carry earth onto the the hilltop shaped like a hemisphere of radius 82 feet, to build a truncated cone with its side tangent to the hemisphere and forming a 45° angle with the horizontal (see the *figure*). Thus the height will exceed 1000 feet and the hill would qualify to be called a mountain. How many cubic feet of earth need to be carried onto the hilltop? **Exercises upwards of grade 11: C. 1446.** Q is a point inside the parallelogram $ABCD$ such that $\angle AQB+\angle CQD=180^\circ$. Prove that $\angle QBA=\angle QDA$ and $\angle QAD=\angle QCD$. **C. 1447.** The Hungarian term for probability theory is "valószínűség-számítás". What is the probability that by selecting and writing down two random characters from each of the words VALÓSZÍNŰSÉG, and SZÁMÍTÁS, the same two-letter string is obtained in both cases?

Problem **C. 1437.** was incorrectly stated in our October issue. Solutions for the corrected problem will be accepted together with those in the November issue.

C. 1437. Each of nine distinct lines divides the area of a square in a ratio 2 : 3 such that no line cuts off a triangle from the square. Prove that three out of the nine lines are concurrent.

New exercises – competition B (see page 478): **B. 4903.** Determine all positive integers a, b, c, d such that $abcd-1\mid a+b+c+d$. (*4 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 4904.** A plane figure S has exactly two axes of symmetry. Show that S has central symmetry, too. (*3 points*) **B. 4905.** Let $a_1\geqslant a_2\geqslant a_3\geqslant\dots\geqslant a_{2n-1}\geqslant a_{2n}\geqslant 0$, and $\sum_{i=1}^{2n}a_i=1$. Prove that $a_1a_2+3a_3a_4+5a_5a_6+\dots+(2n-1)a_{2n-1}a_{2n}\leqslant\frac{1}{4}$. When will the equality hold? (*4 points*) **B. 4906.** The midpoints of sides BC and CD of a convex quadrilateral $ABCD$ are E and F, respectively. The line segments AE, EF and AF divide the quadrilateral into four triangles whose areas are four consecutive integers. What is the maximum possible area of triangle ABD? (*5 points*) (Proposed by *S. Róka,* Nyíregyháza) **B. 4907.** 1×1 squares are placed on a rectangle of dimensions $a\times b$, with sides parallel to the sides of the rectangle. Prove that the maximum number of such squares without overlaps is $[a]\cdot[b]$ (where $[x]$ stands for the greatest integer not greater than the number x). (*5 points*) **B. 4908.** Let C denote an arbitrary point on the circumference of a circle of diameter AB. Let T be the perpendicular projection of point C onto the line segment AB. Draw the circle of centre C that passes through T. Let the intersections of the two circles be P and Q. Prove that line PQ bisects the line segment CT. (*4 points*) (*Kvant*) **B. 4909.** Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that the equation $x\cdot f(y)-y\cdot f(x)=f\left(\frac{y}{x}\right)$ holds for all $x\neq 0$ and y. (*6 points*) (*Kvant*) **B. 4910.** Let P, Q, R and S denote points on the lines of the sides of a square $ABCD$ such that $AP=BQ=CR=DS$, as shown in the *figure.* Starting from an arbitrary interior point X of side AB, let the line PX intersect line BC at Y, let QY intersect line CD at Z, let RZ intersect line DA at V, and finally let SV intersect line AB at X'. Prove that if X' and X coincide then $XYZV$ is a square. (*5 points*) **B. 4911.** We have placed chessmen on a 8×8 chessboard. There is an odd number of chessmen standing in every row, and in every column. Prove that the total number of chessmen on the black fields of the chessboard is even. (*5 points*)

New problems – competition A (see page 480): **A. 707.** 100 betyárs stand on the Hortobágy plains. Every betyár's field of vision is a 100 degree angle. After each of them announces the number of other betyárs they see, we compute the sum of these 100 numbers. What is the largest value this sum can attain? **A. 708.** Let S be a finite set of rational numbers. For each positive integer k, let $b_k=0$ if we can select k (not necessarily distinct) numbers in S whose sum is 0, and $b_k=1$ otherwise. Prove that the binary number $0.b_1b_2b_3\ldots$ is a rational number. Would this statement remain true if we allowed S to be infinite? **A. 709.** Let $a>0$ be a real number. Find the minimal constant C_a for which the inequality $C_a\sum_{k=1}^{n}\frac{1}{x_k-x_{k-1}}>\sum_{k=1}^{n}\frac{k+a}{x_k}$ holds for any positive integer n and any sequence $0=x_0<x_1<\cdots<x_n$ of real numbers.

Problems in Physics

(see page 506)

M. 372. Make a sandglass from a cylinder-shaped plastic (PET) water bottle. Make a small hole (of diameter approximately 8-10 mm) on the bottle cap through which the dry sand can flow out. Measure how the amount of the out-flowing sand in a unit of time depends on the height of the sand in the bottle.

G. 613. A vehicle undergoes circular motion at a constant speed of 72 km/h. How much time elapses until it gets back to the same point if its acceleration is 1.6 m/s^2? **G. 614.** Disks of equal mass of m were attached to the ends of a negligible-mass spring of spring constant D. The disks and the spring in unstretched position are placed to an air-cushioned table, and they are given a velocity of v_0 in the direction of the axis of the spring. At a certain instant the disk at the back is suddenly stopped and held at rest. *a*) How much time elapses until the other disk turns back? *b*) What is the greatest extension of the spring and at most how much potential energy is stored in the spring? *Data:* $D=16$ N/m, $m=0.25$ kg, $v_0=2$ m/s. **G. 615.** A container, filled with water halfway, is sliding down (with some acceleration) along a long enough slope of angle of elevation of 30°. What is the angle between the surface of the water and the plane of the slope, if friction is negligible? **G. 616.** The radius of a thin-walled, negligible-mass gymnastic ball is 30 cm, the pressure of the enclosed air is $1.1\cdot 10^5$ Pa, and the ambient air pressure is $1.0\cdot 10^5$ Pa. How much does the volume of the ball decrease when a 50 kg