Pontversenyen kívüli feladat

A KöMaL pontversenyében kitűzött, szokásos számolási és mérési feladatokon kívül a tudományos és műszaki életben sokszor találkozhatunk olyan problémákkal, amelyek kezeléséhez a fizikai és matematikai ismeretek mellett közgazdasági és jogszabályi információkra, pályázatírási készségre, egymásnak részben ellentmondó feltételek mellett döntéshozatali bátorságra is szükségünk lehet.

Ilyen feladatok közzétételével, azokat megfelelő háttérinformációval kiegészítve megpróbáljuk a KöMaL hagyományos tevékenységi körét bővíteni, „életszerű" helyzetek (esetleírások) ismertetésével és a hozzájuk kapcsolódó feladatokkal ízelítőt nyújtani azon problémák sokszínűségéből, amelyekkel az iskola (egyetem) elvégzése után találkozni fognak Olvasóink.

A feladatok a KöMaL honlapján találhatók meg (`http://www.komal.hu/cikkek/fizika-mtaek/fizika-mtaek.h.shtml`), elektronikusan küldhetők be a `szerk@komal.hu` címre a probléma sorszámának és címének feltüntetésével az ott megjelölt határidőig. Ezek a feladatok nem számítanak bele a pontversenybe, de a megoldásokat értékeljük, és a legjobbakat díjazzuk.

Az első ilyen jellegű probléma: *Elveszett radioaktív sugárforrás megtalálása.* (Beküldési határidő: 2017. november 10. A feladat az MTA Energiatudományi Kutatóközpont támogatásával kerül kitűzésre.)

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Beküldési határidő: 2017. november 10.
Elektronikus munkafüzet: `https://www.komal.hu/munkafuzet`
Cím: KöMaL feladatok, Budapest 112, Pf. 32. 1518

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MATHEMATICAL AND PHYSICAL JOURNAL FOR SECONDARY SCHOOLS
(Volume 67. No. 7. October 2017)

Problems in Mathematics

New exercises for practice – competition K (see page 415): **K. 553.** A bag contains the numbers 1 to 200, written on cards. Andrew and Bill take turns drawing number cards one by one until the bag is empty. At the end, each of them adds his numbers together. Given that the first number drawn by Andrew is 3 and Bill's first number is 170, by what maximum amount may Andrew's sum exceed Bill's sum at the end? **K. 554.** The integers 1 to 2017 are listed as follows: first those numbers not divisible by 3 are written down in increasing order. Then the list continues with those numbers that are divisible by 3 but not divisible by 9, followed by those divisible by 9 but not divisible by 27, and so on. *a*) What is the last number of the list? *b*) In which position will 2017 be in the list? *c*) In which position will 2016 be in the list? **K. 555.** For which three consecutive integers is their product five times their sum? **K. 556.** In a lattice of unit squares, is there a pentagon whose vertices are all lattice points and whose sides are all $\sqrt{5}$ units long? **K. 557.** The midpoints of the sides of a square $ABCD$ are P, Q, R

and S. They are connected to the vertices of the square as shown in the *figure.* Prove that $AT = TV$. **K. 558.** For which positive integers n will $n^4 + n^2 + 1$ be a prime?

New exercises for practice – competition C (see page 416): **Exercises up to grade 10: C. 1434.** In a running race organized in Munich, the participants started simultaneously and ran along a set path. 30 minutes after the start of the race, a car set out from the starting line and followed the runners at uniform speed. For each participant, the race terminated whenever the car caught up with him or her. The female winner was overtaken by the car at 68 km, and the male winner was overtaken at 92 km, 1 hour and 36 minutes later. Assuming that they also ran at uniform speed, what were the speeds of the two winners, and what was the speed of the car? **C. 1435.** Inside a square of side 2 units, semicircles are drawn over two adjacent sides as diameters. What is the radius of the circle that touches one semicircle and the side of the square internally, and touches the other semicircle externally? (Proposed by *D. Fülöp,* Pécs) **Exercises for everyone: C. 1436.** We have eight red cubes and eight white cubes, all congruent. We select eight cubes and form a large cube out of them. How many differently coloured large cubes may we obtain? Two cubes are differently coloured if they cannot be rotated into each other. (*Matlap,* Kolozsvár) **C. 1437.** Given that each of nine distinct lines divides the area of a square in a $2:3$ ratio, prove that there are three concurrent lines among them. **C. 1438.** Prove that the equation $x^2 + y^3 = z^4$ has no solution of prime numbers x, y, z. **Exercises upwards of grade 11: C. 1439.** For what value of c will the simultaneous equations $(x-5)^2+(y-1)^2 = c$, $(x-1)^2+(y-5)^2 = c$ have a unique solution? **C. 1440.** In the unit cube $ABCDA'B'C'D'$, let M and N denote the perpendicular projections of points D' and B onto the diagonal $B'D$ of the cube, respectively. Determine the area of quadrilateral $BND'M$. (*Matlap,* Kolozsvár)

New exercises – competition B (see page 417): **B. 4894.** Seven thieves have stolen some golden coins, and now each of them takes a share of the loot in the following manner. They proceed in alphabetical order of their names, and everyone takes as many coins as the sum of the digits of the number of coins in the heap not distributed yet. The last coin is removed when two full circles are completed. It turns out that everyone has received the same number of coins, only the chief got more. What was the position of the chief in the alphabetical order? (*4 points*) (*Matlap,* Kolozsvár) **B. 4895.** Prove that if $n-1$ and $n+1$ are both primes and $n > 6$ is an integer then $n^2(n^2+16)$ is divisible by 720. (*3 points*) (*English competition problem*) **B. 4896.** Let A_1, B_1, C_1, D_1 denote the midpoints of the sides of a convex quadrilateral $ABCD$. Let A_2, B_2, C_2, D_2 denote the midpoints of the sides of the quadrilateral $A_1B_1C_1D_1$. The procedure is continued. Prove that if quadrilateral $A_1B_1C_1D_1$ is cyclic then quadrilateral $A_{2017}B_{2017}C_{2017}D_{2017}$ is also cyclic. (*3 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 4897.** Given n points in the plane, show that it is possible to select three, denoted by A, B and C, such that $\angle ABC \leqslant 180°/n$. (*4 points*) **B. 4898.** Let A be a four-element set of positive integers such that $ab+13$ is a perfect square for all $a, b \in A$, $a \neq b$. Prove that each element of A leaves a remainder of 2 when divided by 4. (*4 points*) (Proposed by *G. Nyul,* Debrecen) **B. 4899.** The degree of each vertex of a simple planar graph G is 3, and G can be drawn in the plane with its edges represented by non-intersecting unit line segments. Show that G has at least 8 vertices. (*5 points*) **B. 4900.** Let K be a convex shape symmetric in the origin, let e be a line through the origin, and let e' be any line parallel to e. Let, furthermore $\#H$ denote the number of lattice points in a set H. Prove that $\#(K \cap e) + 1 \geqslant \#(K \cap e')$. (*5 points*) **B. 4901.** An epidemic broke out in Smurf village after a few residents contracted a disease. Luckily, every smurf recovers from the illness in one day, and then they will be immune to the disease for another day. However, from the following day onwards they may catch the disease again. The transition between the sick and healthy states always occurs at night when the smurfs are sleeping. Unfortunately, the smurfs never give up their habit of visiting all their friends every day, not even when they are ill. So when a sick smurf meets a healthy but not immune one, the latter will inevitably catch the disease. Given that Smurf village has 100 inhabitants, show that the epidemic will necessarily terminate by the 101th day following the outbreak. (*6 points*) (Collected at the *Budapest Univ. Technology and Economics*) **B. 4902.** Let A_1B_1, A_2B_2, A_3B_3 and A_4B_4 be four parallel line segments of different lengths given in the plane. For any $1 \leqslant i < j \leqslant 4$, let M_{ij} denote the intersection of lines A_iB_j and A_jB_i. Show that the lines $M_{12}M_{34}$, $M_{13}M_{24}$ and $M_{14}M_{23}$ are either concurrent or parallel. (*6 points*)

New problems – competition A (see page 419): **A. 704.** A regular triangle has side length n. We divided its sides into n equal parts and drew a line segment parallel with each side through the dividing points. A lattice of $1+2+\dots+(n+1)$ intersection points is thus formed. For which positive integers n can this lattice be partitioned into triplets of points which are the vertices of a regular triangle of side length 1? (Proposed by *Alexander Gunning,* Cambridge, UK) **A. 705.** Triangle ABC has orthocenter H. Let D be a point distinct from the vertices on the circumcircle of ABC. Suppose that circle BHD meets AB at $P \neq B$, and circle CHD meets AC at $Q \neq C$. Prove that as D moves on the circumcircle, the reflection of D across line PQ also moves on a fixed circle. (Proposed by *Michael Ren,* Andover, Massachusetts, USA) **A. 706.** Let $\mathbb{Z}^+$ denote the set of positive integers. Find all functions $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ which satisfy the following: $f(mn) = f(m)f(n)$ for all $m, n \in \mathbb{Z}^+$, and $f^{(n)}(n) = n$ for all $n \in \mathbb{Z}^+$ (in other words, $f\Big(f\big(\dots\big(f(n)\big)\dots\big)\Big) = n$, where there are n pairs of brackets on the left-hand side). (*Korean problem*)

Problems in Physics

(see page 441)

M. 371. Suspend two alike AA battery bifilarly and make them collide, such that they move along their longer symmetry axis, and collide with their negative terminals, which is on their flatter sides. Determine the *coefficient of restitution* (which is the ratio of the relative velocity after collision to that of before collision). Carry out the measurement with two new batteries, with a new one and a discharged one, and with two discharged ones.

G. 609. A man leans against the wall of a house in a peculiar way as shown in the *figure,* and exerts a force of F onto the wall. If he is observed from the reference frame of the ground, the man does not perform work, because his displacement is zero. According to another observer who is travelling in a car, moving at a speed of v, the man exerts a constant force while moving a long distance, so he does work. Why doesn't the man leaning against the house get exhausted? **G. 610.** A meteorite of mass m, of specific heat capacity c and of specific latent heat of fusion L, consists of some material which is extremely good heat conductor. When it reaches the atmosphere of the Earth the temperature of the meteorite is ΔT below its melting point. In the atmosphere due to friction heat is generated in it at a rate of P. How long does it take for the meteorite to be melted totally? **G. 611.** What is the area of a floating ice floe of thickness 30 cm if it can hold an 80 kg man? **G. 612.** From the window of a house close to the M3 motorway stretch, starting at Budapest, we observe a Boeing 737 – ready to land at Ferihegy – flying above us. A crow permanently follows the plane (seemingly the "bird flies next to the plane"), and the wingspan of the crow seems to have the same length as the length of